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Public debt in an OLG model with imperfect competition:

Long-run effects of austerity programs and changes in the growth rate

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Abstract
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Public debt in an OLG model with imperfect competition: long-run effects of austerity programs and changes in the growth rate *

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Abstract

We show that (i) dynamic inefficiency may be empirically relevant in a modified Diamond model with imperfect competition, (ii) if fiscal policy is used to avoid inefficiency and maintain an optimal capital intensity, the required debt ratio will be inversely related to the growth rate, and (iii) austerity policies – reductions in government consumption and entitlement programs for the old generation – raise the required debt ratio.

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1 Introduction

Diamond (1965) provides a classic analysis of public debt in dynamically inefficient economies. His well-known results gain real-world significance insofar as actual economies become dynamically inefficient in the absence of public debt. The general consensus seems to be that this is not the case: empirically the rate of return on capital appears to exceed the rate of growth.

The empirical argument has two potential weaknesses. To ascertain the need for fiscal policy and public debt one would need to evaluate the rate of return in a state without public debt; it is not sufficient to show that dynamic efficiency may hold if the evidence applies to an economy with significant amounts of public debt. Second, the standard efficiency criterion – that the rate of return exceed the growth rate – is based on the identification of the rate of return with the marginal product of capital. This identification breaks down under imperfect competition. Dynamic inefficiency, as a result, may be empirically relevant.

Our analysis is motivated by the recent focus on public debt in policy debates. Some of the striking findings in Reinhart and Rogoff (2010) have been discredited (Herndon et al. 2013) and claims that high debt reduces economic growth have been challenged by a number of studies (e.g. Irons and Bivens (2010), Dube (2013), Basu (2013)). But the challenges have been largely empirical. Our analysis contributes a theoretical perspective. We find a relationship between debt and growth rates, but the causal link unambiguously runs from growth to debt: a low growth rate generates a high steady-growth ratio of debt to income if fiscal policy is used to avoid dynamic inefficiency and maintain an optimal capital intensity. The analysis also shows that austerity policies have paradoxical effects: reductions in government consumption and in entitlement programs for the old generation raise the long-run debt ratio.

OLG models with imperfect competition have been developed by, among others, d'Aspremont et al. (1995), Pagano (1990) and Jacobsen and Schultz (1994). Like our analysis in this paper, these models examine the potential usefulness of fiscal policy. But this similarity masks fundamental differences in the structure of the models and the nature of the fiscal effects. Assuming Cournot competition, d'Aspremont et al. show that fiscal policy can be used to influence the equilibrium markup which – along with an elastic labor supply – determines equilibrium employment; the model has no capital and no dynamic inefficiency, and the government balances its budget in each period. The details

are different in Pagano and Jacobsen and Schultz, but the fiscal effects run through changes in competition and market power in these papers too. By contrast, the markup is constant in our setting, and we treat the labor supply as inelastic; our focus is on dynamic efficiency and the dynamics of public debt.[1]

Public debt dynamics have been examined in an OLG setting by Chalk (2000). Assuming a constant primary deficit per worker, he finds that even if the economy is dynamically inefficient when public debt is zero, a constant primary deficit may be unsustainable. Moreover, in those cases where a primary deficit is sustainable, convergence is to a steady growth path that is dynamically inefficient. These results invite several questions. Why would a government want to pursue policies of this kind? Why focus on trajectories that keep a constant primary deficit? Economic analysis of monetary policy typically looks for optimal policies (or policy rules), given some welfare function and a model of how the economy operates. Our analysis of fiscal policy is in a similar spirit.[2]

Section 2 presents the basic model with imperfect competition. Taxation and public debt are added in section 3. Section 4 discusses implications of the analysis, and section 5 offers some concluding remarks.

2 An OLG model with imperfect competition

Production There is a continuum of young agents indexed by j with $j \in [0, L_t]$. The population grows at the rate n, $L_{t+1} = (1+n)L_t$. Agents live for two periods: they work in the first period and live off their savings in the second. In addition to a labor endowment, each agent has the know-how to produce a particular intermediate good using capital and labor. A CES production function describes the production of the intermediate goods y_{jt}:

$$y_{jt} = [\alpha(\sigma k_{jt})^{\gamma} + (1-\alpha)(\lambda l_{jt})^{\gamma}]^{1/\gamma}, \quad 0 < \alpha < 1,\ \gamma < 1 \tag{1}$$

where k_{jt} and l_{jt} denote the inputs of capital and labor in the production of good j at time t.

[1] Bohn (2009) examines fiscal policy in relation to another source of market failure: inefficiencies associated with productivity shocks and intergenerational risk. He assumes a balanced budget, and leaves out imperfect competition and problems of dynamic inefficiency. In this paper we ignore stochastic shocks and risk.

[2] Although Keynesian aggregate demand problems are excluded in this paper, the approach has affinities with the Keynesian literature on 'functional finance' (Lerner 1943). Recent contributions to this literature include Schlicht (2006), Godley and Lavoie (2007), Arestis and Sawyer (2010), Kregel (2010), Palley (2010), Ryoo and Skott (2013), and Costa Lima et al. (2013).

Intermediate goods are used to produce final output (Y_t):

$$Y_t = \left[A_t \int_0^{L_t} y_{jt}^{1-\frac{1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1 \tag{2}$$

where ε is the elasticity of substitution among intermediate goods. A growing population – a growing number of intermediate inputs – would be a source of productivity increases and growth in per capita income if the multiplicative term A_t were constant. Important as they may be, these issues are not the concerns of this paper; to simplify, we therefore assume that

$$A_t = L_t^{-\frac{1}{\varepsilon}} \tag{3}$$

With this specification, population growth will have no direct effect on productivity.

The final good can be used for consumption or transformed (costlessly) into capital for use in the production of intermediate goods. There is perfect competition in the final goods market and firms maximize profits,

$$\max_{y_{jt}} \quad p_t Y_t - \int_0^{L_t} p_{jt} y_{jt} dj \tag{4}$$

where p_t and p_{jt} are the prices of final output and intermediate goods.

The first-order condition yields

$$\frac{y_{jt}}{y_{\kappa t}} = \left(\frac{p_{jt}}{p_{\kappa t}} \right)^{-\varepsilon}, \quad \forall j, \kappa \in [0, L_t] \tag{5}$$

which, along with (2) and a zero-profit condition, implies that

$$y_{jt} = \left(\frac{p_{jt}}{p_t} \right)^{-\varepsilon} \frac{Y_t}{L_t} \tag{6}$$

and

$$p_t = \left[\frac{1}{L_t} \int_0^{L_t} p_{jt}^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \tag{7}$$

Using the final good as numeraire, we assume $p_t = 1$ for all t.

Intermediate good producers maximize profits subject to the isoelastic de-

mand function (6) and the production function (1):

$$\begin{aligned} \max_{k_{jt},\ l_{jt}} \quad & p_{jt}y_{jt} - w_t l_{jt} - (r_t + \delta)k_{jt} \\ s.t. \quad & y_{jt} = [\alpha(\sigma k_{jt})^{\gamma} + (1-\alpha)(\lambda l_{jt})^{\gamma}]^{1/\gamma} \\ & p_{jt} = p_t \left(\frac{Y_t}{L_t}\right)^{1/\varepsilon} y_{jt}^{-1/\varepsilon} \end{aligned} \tag{8}$$

where w_t, r_t and δ are the real wage, the real interest rate and the rate of capital depreciation. The first-order conditions give a mark-up equation:

$$p_{jt} = \frac{\varepsilon}{\varepsilon - 1} c_t \tag{9}$$

where c_t represents the unit cost:

$$c_t \equiv \left[\alpha^{\frac{1}{1-\gamma}} \sigma^{\frac{\gamma}{1-\gamma}} (r_t + \delta)^{\frac{-\gamma}{1-\gamma}} + (1-\alpha)^{\frac{1}{1-\gamma}} \lambda^{\frac{\gamma}{1-\gamma}} w_t^{\frac{-\gamma}{1-\gamma}}\right]^{-\frac{1-\gamma}{\gamma}} \tag{10}$$

The unit cost is the same for all input producers. Thus, p_{jt} is the same for all j, and, using (7) and (9),

$$1 = p_t = p_{jt} = \frac{\varepsilon}{\varepsilon - 1} c_t \tag{11}$$

Relative factor demands are given by

$$k_{jt} = \left[\frac{\alpha}{1-\alpha} \frac{\sigma^{\gamma}}{\lambda^{\gamma}} \frac{w_t}{(r_t + \delta)}\right]^{\frac{1}{1-\gamma}} l_{jt} \tag{12}$$

and integrating (12) over $[0, L_t]$, the aggregate capital stock K_t satisfies

$$K_t = \left[\frac{\alpha}{1-\alpha} \frac{\sigma^{\gamma}}{\lambda^{\gamma}} \frac{w_t}{(r_t + \delta)}\right]^{\frac{1}{1-\gamma}} L_t \tag{13}$$

Equations (10) and (11) give us a factor-price frontier; we can write the real wage as a function of the interest rate:

$$w_t = (1-\alpha)^{\frac{1}{\gamma}} \lambda \left\{ \left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\frac{\gamma}{1-\gamma}} - \alpha^{\frac{1}{1-\gamma}} \sigma^{\frac{\gamma}{1-\gamma}} (r_t + \delta)^{\frac{-\gamma}{1-\gamma}} \right\}^{-\frac{1-\gamma}{\gamma}} \equiv w(r_t) \tag{14}$$

Saving There is full employment and we take the labor supply to be inelastic.[3] Normalizing the supply of an individual worker to one, the budget constraint is given by

$$c_{1,t} + \frac{1}{1+r_{t+1}} c_{2,t+1} = w_t + \pi_t \tag{15}$$

where $c_{1,t}$ and $c_{2,t+1}$ are the levels of consumption when the agent is young and old; π_t is the agent's profits from production of an intermediate good. Using (9)-(12), we have

$$\pi_t = \frac{1}{\varepsilon - 1} \frac{w(r_t)}{\eta(r_t)} \equiv \pi(r_t) \tag{16}$$

where $\eta(r_t)$ is the share of labor in unit cost:

$$\begin{aligned} \eta(r_t) &\equiv \frac{w_t L_t}{w_t L_t + (r_t + \delta) K_t} \\ &= \frac{(1-\alpha)^{\frac{1}{1-\gamma}} \lambda^{\frac{\gamma}{1-\gamma}} w(r_t)^{\frac{-\gamma}{1-\gamma}}}{(1-\alpha)^{\frac{1}{1-\gamma}} \lambda^{\frac{\gamma}{1-\gamma}} w(r_t)^{\frac{-\gamma}{1-\gamma}} + \alpha^{\frac{1}{1-\gamma}} \sigma^{\frac{\gamma}{1-\gamma}} (r_t + \delta)^{\frac{-\gamma}{1-\gamma}}} \end{aligned} \tag{17}$$

The utility function for a young agent in period t is commonly specified using a general CIES form,

$$U_t = \frac{c_{1,t}^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho} \frac{c_{2,t+1}^{1-\theta} - 1}{1-\theta}; \qquad \theta \geq 0 \tag{18}$$

where θ and ρ are the inverse of the intertemporal elasticity of substitution and the discount rate. Using this specification, the maximization problem yields an expression for optimal consumption:

$$c_{1,t} = (1 - s_t)(w_t + \pi_t) \tag{19}$$

where the young generation's saving rate s_t can be written

$$s_t = \frac{[(1+r_{t+1})]^{(1-\theta)/\theta}}{(1+\rho)^{1/\theta} + [(1+r_{t+1})]^{(1-\theta)/\theta}} \tag{20}$$

In the logarithmic case ($\theta \to 1$ and $U_t = \log c_{1,t} + \frac{1}{1+\rho} \log c_{2,t+1}$) this expression simplifies to

$$s_t = s = \frac{1}{2+\rho} \tag{21}$$

Given the purpose of this paper – to examine complications from imperfect

[3] Lopez-Garcia (2008) provides an extension with an endogenous labor supply.

competition – we keep the saving side simple and focus on the logarithmic case ($\theta \to 1$, $s_t = s$).

The capital stock in period $t+1$ is determined by the young generation's saving in period t (S_t):

$$K_{t+1} = S_t = s(w_t + \pi_t)L_t \tag{22}$$

Using (14) and (16), equation (22) can be written as a first-order difference equation of the interest rate.

$$\left[\frac{\alpha}{1-\alpha}\frac{\sigma^\gamma}{\lambda^\gamma}\frac{w(r_{t+1})}{(r_{t+1}+\delta)}\right]^{\frac{1}{1-\gamma}}(1+n) = s[w(r_t)+\pi(r_t)] \tag{23}$$

Equation (23) fully determines the trajectory of the interest rate.

Steady states In a steady state we have $r_t = r_{t+1} = r$ for all t and, substituting for $w(r)$ and $\pi(r)$, equation (23) can be re-written

$$\begin{aligned} 1+n &= s(w+\pi)L/K \\ &= s\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\gamma}}\alpha^{-\frac{1}{1-\gamma}}\sigma^{-\frac{\gamma}{1-\gamma}}(r+\delta)^{\frac{1}{1-\gamma}} - (r+\delta)\right] \end{aligned} \tag{24}$$

As shown in Appendix A,

1. in the case of $0 \leq \gamma < 1$, equation (24) has a unique root $r \in (-\delta, \infty)$. This fixed point is stable.

2. in the case of $\gamma < 0$, equation (24) has two distinct roots $r \in (-\delta, \infty)$ if σ is sufficiently large (given the values of the other parameters). Intuitively, if σ is too low, it becomes impossible to accumulate capital at the required rate n for any non-negative level of consumption, and the equation has no solution (cf. the Leontief example below). The smaller root for r is (locally) stable; the larger root is unstable. The large solution may require a negative real wage rate and thus may not be meaningful, even disregarding stability questions.

The term on the right-hand side of (24) – $(w+\pi)L/K$ – represents the young agents' income normalized by the capital stock. This income-capital ratio is increasing in the interest rate in the neighborhood of stable steady states. This property can be used for comparative statics. First, an increase in the growth

rate n or a fall in the saving rate s (a rise in the discount rate ρ) requires an increase in the income-capital ratio in order to maintain a steady state. Thus, the interest rate has to go up. The equilibrium interest rate, second, is inversely related to the degree of market power. An increase in the mark-up factor – a reduction in the elasticity of substitution among intermediate goods (ε) – raises the income-capital ratio for a given interest rate; the real interest rate therefore needs to be lowered to keep the income-capital ratio constant at $(1+n)/s$.

Example: the Leontief case If $\gamma \to -\infty$, the production function converges to the Leontief form,

$$y_{jt} = \min\{\sigma k_{jt}, \lambda l_{jt}\} \tag{25}$$

In order for full-employment growth to be technically feasible, the aggregate capital stock must grow at least as fast as the labor force when all output is being invested. Algebraically,

$$\sigma K_t \geq Y_t \geq (n+\delta)K_t$$

or

$$\sigma \geq n + \delta \tag{26}$$

This technical feasibility condition is necessary but not sufficient. With a logarithmic utility function and a saving rate of $1/(2+\rho)$, the parameters need to satisfy the more restrictive condition

$$sY_t = \frac{Y_t}{2+\rho} \geq K_{t+1} = (1+n)K_t \geq \frac{1+n}{\sigma}Y_t$$

or

$$\sigma \geq (1+n)(2+\rho) \tag{27}$$

The economy has two steady growth paths. There is a full-utilization path with $\sigma k_{jt} = \lambda l_{jt} = y_{jt}$, $K_t = (\lambda/\sigma)L_t$ and $c_t = \frac{w_t}{\lambda} + \frac{r_t+\delta}{\sigma}$. Along this path the pricing equation (11) takes the form $1 = p_t = p_{jt} = \frac{\varepsilon}{\varepsilon-1}\left(\frac{w_t}{\lambda} + \frac{r_t+\delta}{\sigma}\right)$; the real wage and the amount of profits are given by $w_t = \lambda\left(1-\frac{1}{\varepsilon}\right) - \lambda\frac{r_t+\delta}{\sigma}$ and $\pi_t = \frac{\lambda}{\varepsilon}$, respectively. Using these expressions, equation (22) can be written as

$$\begin{aligned} \frac{K_{t+1}}{L_t} &= \frac{K_{t+1}}{L_{t+1}}(1+n) \\ &= \frac{\lambda}{\sigma}(1+n) = s(w_t+\pi_t) = s\left[\lambda - \lambda\frac{r_t+\delta}{\sigma}\right] \end{aligned} \tag{28}$$

Solving (28) for the rate of interest and substituting the result back into the factor-price frontier, we have:[4]

$$r + \delta = \sigma - (2 + \rho)(1 + n) > 0 \tag{29}$$

$$w = \frac{\lambda(2 + \rho)(1 + n)}{\sigma} - \frac{\lambda}{\varepsilon} \tag{30}$$

The steady growth path described by equations (29)-(30) is dynamically efficient: the net marginal product associated with a reduction in the capital-labor ratio exceeds the growth in the labor force ($\sigma - \delta > n$). This high-interest path is unstable, however. Starting from the efficient path, a positive shock to w raises saving and capital intensity increases in the next period to give $K_{t+1}/L_{t+1} > \lambda/\sigma$. The young workers' income (the wage rate plus profits) then rises to λ in subsequent periods[5], and the economy will be following a steady growth path with excess capacity:

$$\frac{K}{L} = \frac{\lambda}{(2 + \rho)(1 + n)} > \frac{\lambda}{\sigma} \tag{31}$$

This steady-growth path is dynamically inefficient; the net marginal product of capital is equal to $-\delta < n$; consumption could be increased by reducing investment (eliminating the excess capacity) and having each young generation use some of its saving to finance consumption for the old generation.

Dynamic efficiency Returning to the general case ($-\infty < \gamma \leq 1$), the general condition for efficiency is for the net marginal product of capital to exceed the growth rate of the labor force. Under perfect competition, this criterion can be re-stated as $r > n$. The introduction of imperfect competition modifies the efficiency criterion. Imperfect competition, first, drives a wedge between the marginal product of capital and the interest rate. With our specification, the marginal product of capital equals $\frac{1}{c}(r + \delta) = \frac{\varepsilon}{\varepsilon - 1}(r + \delta)$, and the efficiency criterion can be written

$$\frac{\varepsilon}{\varepsilon - 1}(r + \delta) - \delta > n$$

[4] The solution for w becomes negative and non-meaningful if the monopoly rents are too large (a small ε) and/or the capital productivity is too large (a large σ). This is quite intuitive: under these conditions the saving decisions of the young generation will generate a growth rate of the capital stock that exceeds n even when the wage rate is zero.

[5] The wage rate and the amount of profits will be $w_t = \lambda\left(1 - \frac{1}{\varepsilon}\right)$ and $\pi_t = \frac{\lambda}{\varepsilon}$ with $w_t + \pi_t = \lambda$.

or

$$r > \left(\frac{\varepsilon - 1}{\varepsilon}\right)(n + \delta) - \delta \tag{32}$$

The wedge implies that $r > n$ is sufficient but not necessary for dynamic efficiency; imperfect competition relaxes the condition for dynamic efficiency *for any given value of* r. But the mark-up factor $\varepsilon/(\varepsilon - 1)$ influences r. More specifically, an increase in the markup decreases r in stable steady states and, using equation (24), it is readily seen that a decrease in ε (corresponding to a rise in the markup) must also lead to a decline in the equilibrium value of the marginal product of capital.[6] Thus, an increase in market power tends to tighten the condition for dynamic efficiency.

More importantly, biases arise if the marginal product of capital is being evaluated by looking at the ratio of gross profits $(pY - wL)$ to capital: under imperfect competition some of the profits are monopoly rents that have no relation to marginal productivity.[7] The bias is particularly clear in the simple Leontief case: producers with excess capacity receive positive gross profits even though the gross marginal product is zero. More generally, using the model in

[6] Re-stating equation (24) we have

$$1 + n = s\left[\left(\frac{\varepsilon}{\varepsilon - 1}\right)^{\frac{1}{1-\gamma}} \alpha^{-\frac{1}{1-\gamma}} \sigma^{-\frac{\gamma}{1-\gamma}} (r + \delta)^{\frac{1}{1-\gamma}} - (r + \delta)\right]$$

By the implicit function theorem,

$$\frac{d(r+\delta)}{d\left(\frac{\varepsilon}{\varepsilon-1}\right)} = -\frac{\frac{1}{1-\gamma}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\gamma}-1}(r+\delta)^{\frac{1}{1-\gamma}}}{\frac{1}{1-\gamma}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\gamma}}(r+\delta)^{\frac{1}{1-\gamma}-1} - \alpha^{\frac{1}{1-\gamma}}\sigma^{\frac{\gamma}{1-\gamma}}} < 0$$

Hence,

$$\begin{aligned}\frac{d\left[\frac{\varepsilon}{\varepsilon-1}(r+\delta)\right]}{d\left(\frac{\varepsilon}{\varepsilon-1}\right)} &= r + \delta + \frac{\varepsilon}{\varepsilon - 1}\frac{d(r+\delta)}{d\left(\frac{\varepsilon}{\varepsilon-1}\right)} \\ &= -(r+\delta)\frac{\alpha^{\frac{1}{1-\gamma}}\sigma^{\frac{\gamma}{1-\gamma}}(r+\delta)}{\frac{1}{1-\gamma}\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\gamma}}(r+\delta)^{\frac{1}{1-\gamma}} - \alpha^{\frac{1}{1-\gamma}}\sigma^{\frac{\gamma}{1-\gamma}}(r+\delta)} < 0\end{aligned}$$

[7] Abel et al. (1989) use the ratio $(Y - wL)/K$ in their empirical evaluation of dynamic efficiency. They acknowledge that their analysis "depends on the assumption that capital receives its marginal product, an assumption that excludes the possibility that capital income includes substantial monopoly profit" (p.7). They suggest, however, that their calculation is not badly distorted by monopoly profits. In support of this view they point to average values of about one for Tobin's q, arguing that "if a large part of profits reflected returns to something other than physical capital, one would suspect that the firm's market value would substantially exceed the value of their physical assets." (p.10) This faith in Tobin's q as a good indicator of the importance of monopoly profits seems questionable.

this section, we have the following inequality:[8]

$$\frac{\partial Y}{\partial K} < \frac{\partial Y}{\partial K} + \frac{\eta}{1-\eta}\frac{r+\delta}{\varepsilon - 1} = \frac{Y - wL}{K} \tag{33}$$

The interpretation of (33) is intuitive. The rate of gross profits is the sum of $r+\delta$ and the ratio of the entire pure profit (profit margins over *the sum* of interest cost *and wage cost*) to capital; the marginal product of capital, on the other hand, is the sum of $r + \delta$ and a fraction of the ratio of pure profits to capital: only the profit margin on the cost of capital is included. Thus, the upward bias comes from the profit margin on wage cost ($\frac{\eta}{1-\eta}\frac{r+\delta}{\varepsilon-1} = \frac{1}{\varepsilon-1}\frac{w_t}{k_t}$) which is included in the rate of gross profits but not in the marginal product of capital.

The upward bias from the use of measures based on $(Y - wL)/K$ can be significant. As an example, let $\eta = \frac{wL}{wL+(r+\delta)K} = 2/3$, $\varepsilon = 5$ and $r + \delta = 0.08$.[9] With these values, the ratio of (annual) gross profits to capital exceeds the (annual) gross marginal product by four percentage points.

3 Public debt

Extending the model, we introduce a government that consumes (G_t), levies lumpsum taxes on the young and old generations (T_t^Y and T_t^O) and has debt (B_t).[10] Young households save in the form of fixed capital and government bonds; these assets, we assume, are perfect substitutes and have the same rate of return, r_t.

Equation (22) now takes the form

$$K_{t+1} + B_{t+1} = S_t \tag{34}$$

[8] We have

$$\begin{aligned}\frac{Y - wL}{K} &= \frac{Y}{wL + (r+\delta)K}\frac{wL + (r+\delta)K}{K} - \frac{\eta}{1-\eta}(r+\delta)\\ &= \left(\frac{\varepsilon}{\varepsilon - 1}\frac{1}{1-\eta} - \frac{\eta}{1-\eta}\right)(r+\delta)\\ &= \frac{r+\delta}{\varepsilon - 1}\frac{\eta}{1-\eta} + \frac{\partial Y}{\partial K}\end{aligned}$$

[9] The numerical value of the marginal product of capital – like the output-capital ratio, the wage rate w, the rate of depreciation δ, and the rate of return r – depends on the length of the accounting period. The numbers refer to annualized rates.

[10] Since the labor supply is taken to be inelastic, the lumpsum assumption only matters for the old generation.

while the public sector budget constraint is given by

$$G_t + (1 + r_t)B_t = B_{t+1} + T_t^Y + T_t^O \tag{35}$$

The young generation in period t maximizes utility subject to a modified constraint,

$$c_{1,t} + \frac{1}{1 + r_{t+1}} c_{2,t+1} = w_t + \pi_t - \tau_t - \frac{1 + n}{1 + r_{t+1}} \phi_{t+1} \tag{36}$$

where $\tau_t \equiv T_t^Y / L_t$ and $\phi_t \equiv T_t^O / L_t$. This gives the following solution for saving

$$S_t = \left[s(w_t + \pi_t - \tau_t) + (1 - s) \frac{1 + n}{1 + r_{t+1}} \phi_{t+1} \right] L_t \tag{37}$$

Substituting (37) into (34) and dividing through by L_t, equations (34) and (35) can be rewritten,

$$(1 + n)(k_{t+1} + b_{t+1}) = s(w_t + \pi_t - \tau_t) + (1 - s) \frac{1 + n}{1 + r_{t+1}} \phi_{t+1} \tag{38}$$

$$g_t + (1 + r_t)b_t = (1 + n)b_{t+1} + \tau_t + \phi_t \tag{39}$$

where $g_t \equiv G_t / L_t$ and $b_t \equiv B_t / L_t$.

Fiscal policy clearly affects the outcome but that leaves open the determination of an appropriate policy. For present purposes it may be reasonable to assume – somewhat heroically – that government consumption has been set at an optimal level, $g_t = g$. This leaves the two tax parameters, τ_t and ϕ_t. The achievement of dynamic efficiency is one obvious criterion for deciding these tax rates. To be more specific, assume that the capital intensity associated with the interest rate r^* is considered socially optimal in a steady state.[11] One can now look for combinations of τ_t and ϕ_t that maintain $r_t = r^*$.

Taking taxes on the old as exogenous and constant ($\phi_{t+1} = \phi_t = \phi$), equations (38) and (39) give us the required trajectory of the debt ratio:

$$b_{t+1} = a_0 - a_1 b_t \tag{40}$$

[11] As a special case, a social planner could aim for the Golden Rule with the gross marginal product of capital equal to $\delta + n$. The argument, however, is independent of how the optimal capital intensity (the optimal rate of return) is being determined.

where

$$\begin{aligned} a_0 &\equiv \frac{s[w(r^*)+\pi(r^*)-g]-(1+n)k(r^*)+\frac{1+n+s(r^*-n)}{(1+r^*)}\phi}{(1-s)(1+n)} \\ a_1 &\equiv \left(\frac{s}{1-s}\right)\left(\frac{1+r^*}{1+n}\right) \end{aligned}$$

and where $w(r^*), \pi(r^*)$ and $k(r^*) \equiv K/L$ are determined by equations (14), (16) and (13).[12] Solving (40) for the steady state value of b_t, we have:

$$b^* = \frac{a_0}{1+a_1} = \frac{s[w(r^*)+\pi(r^*)-g]-(1+n)k(r^*)}{1+n+s(r^*-n)} + \frac{1}{1+r^*}\phi \qquad (41)$$

By assumption, the steady state is socially optimal and therefore dynamically efficient. The value of b^* can be positive or negative, depending on parameters. In cases where $b^* > 0$, however, an attempt to eliminate the debt could lead to dynamic inefficiency. Putting it differently, the efficiency properties of a hypothetical no-debt economy cannot be ascertained simply by observing that with public debt the economy is in fact efficient. To get some idea of the size of the bias, assume that the initial share of government bonds in total household wealth (capital plus government bonds) is 25%. With this initial composition of household wealth, the elimination of public debt will reduce the gross marginal product by about 25 percent if the production function is Cobb-Douglas;,with plausible values of the initial (annualized) marginal product, this implies that the elimination of public debt reduces the marginal product by 2-3 percentage points.[13] The reduction in the marginal product will be larger (smaller) if the elasticity of substitution is below (above) one.

It should be noted, perhaps, that the designation of a socially optimal capital intensity does not depend on the presence of imperfect competition; the

[12] The difference equation (40) is stable if $a_1 < 1$. The stability condition will be met if the saving rate s is sufficiently low (i.e., the discount rate ρ is sufficiently large). The stability properties of the equation are of limited interest, however: starting from some arbitrary initial values of k_0 and b_0, it will not, in general, be optimal to jump directly to the steady-state value $k(r^*)$ in period 1. The precise design of an optimal policy depends on the details of the social welfare function.

[13] With a Cobb-Douglas production function, $Y = K^\alpha L^{1-\alpha}$, we have

$$\frac{\partial Y}{\partial K} = \alpha K^{\alpha-1} L^{1-\alpha}$$

and

$$\Delta \log \frac{\partial Y}{\partial K} = (\alpha - 1)\Delta \log k$$

The effect on K of eliminating debt follows from the saving equation. If $x = (K+B)/K$ and

expression in equation (41) holds for the any value of ε, including the special case of perfect competition when $\varepsilon \to \infty$. Putting it differently, the value of ε – the degree of imperfection – influences the precise values of r^* and b^* but not the general approach to determining these values.

4 Implications

Debt and growth Given the current focus on possible dangers of public debt, it is interesting to note that the required debt is inversely related to the growth rate n. Differentiating (41) with respect to n, we have

$$\frac{\partial b^*}{\partial n} = -s\frac{(1+r^*)k(r^*) + (1-s)[w(r^*) + \pi(r^*) - g]}{[1+n+s(r^*-n)]^2} < 0 \tag{42}$$

The sum of wage income and monopoly profits exceeds government consumption in actual data as well as for any plausible optimal path; thus $w(r^*) + \pi(r^*) > g$. This inequality implies that $\partial b^*/\partial n < 0$.

The inverse relation between growth and government debt is quite intuitive. The reason for the debt is that the young generation wants to save 'too much'.

$k = K/L$, equation (38) implies that in a steady state

$$\begin{aligned} xk &= \frac{s}{1+n}(w + \pi - \tau) + (1-s)\frac{1}{1+r}\phi \\ &= \frac{s}{1+n}\left(1 - \alpha + \frac{\alpha}{\varepsilon}\right)k^\alpha - \frac{s}{1+n}\tau + (1-s)\frac{1}{1+r}\phi \end{aligned} \tag{*}$$

Assume that the values of g and ϕ are kept unchanged. If the optimal steady state (associated with the initial debt-capital ratio of 1/3) has $r^* = n$, it follows from (39) that the value of τ in the zero-debt steady state will also be unchanged, compared to its value in the optimal steady state. Using (*) and assuming (plausibly) that $\phi \geq 0$ and $\frac{s}{1+n}\tau \geq (1-s)\frac{1}{1+r}\phi$, we now have

$$\frac{d\log\left[\frac{s}{1+n}\left(1-\alpha+\frac{\alpha}{\varepsilon}\right)k^\alpha - \frac{s}{1+n}\tau + (1-s)\frac{1}{1+r}\phi\right]}{d\log k} > \alpha$$

Hence,

$$-(1-\alpha)\Delta\log k < \Delta\log x$$

and

$$\begin{aligned} \Delta\log\frac{\partial Y}{\partial K} &= -(1-\alpha)\Delta\log k \\ &< \Delta\log x \\ &= \log 1 - \log\frac{4}{3} \end{aligned}$$

It follows that

$$\log\frac{\partial Y}{\partial K}^{\text{no debt}} < \log\left(\frac{3}{4}\frac{\partial Y}{\partial K}^{\text{with debt}}\right)$$

But the threshold defining 'too much' depends on the growth rate: a higher growth rate implies that more fixed capital will be needed to employ the future generation and, consequently, that the required amount of public debt will be lower.

Austerity and debt Austerity programs typically include cuts in government consumption and reductions in entitlements like social security or medicare. These austerity measures can be distributionally regressive. Surprisingly, perhaps, from a long-run perspective they are also counterproductive on their own terms: they aggravate the 'debt problem'.

It is readily seen – using (41) – that the required debt (b) depends *inversely* on public consumption (g) and *directly* on the level of taxes on the old generation (ϕ).[14] An increase in g implies that consumption has to contract in order to maintain equilibrium in the product market. With a given ϕ, this is achieved by increasing taxes on the young. As a result the desired saving decreases; this, in turn, reduces the need for government debt as an outlet for saving. Analogously, with a given value of g, an increase in ϕ (corresponding to a reduction in entitlement programs for the old) must be accompanied by a reduction in τ in order to maintain the level of consumption and equilibrium in the goods market; the disposable income of the young increases, and the amount of public debt must increase to meet the rise in saving.

Debt and taxes The debt ratio and the tax on the young are both endogenous, and the long-run correlation between them, not surprisingly,is ambiguous: it depends on the underlying shifts in exogenous variables or parameters. Equations (39) and (41) can be used to derive the steady-growth solution for the tax rate τ :

$$\tau^* = \frac{(r^* - n)[s(w^* + \pi^*) - (1+n)k^*]}{1 + n + s(r^* - n)} - \frac{1+n}{1+r^*}\phi + \frac{1+n}{1+n+s(r^*-n)}g \quad (43)$$

Thus, an increase in g raises τ and reduces b; an increase in ϕ reduces τ and raises b. It follows that shifts in g or ϕ produce a negative correlation between the steady-growth values of τ and b : high debt is associated with low taxes. But other parameter shifts yield a positive correlation; a fall in α, for instance, will raise the wage share and produce an increase in both b and τ if $r^* > n$.

[14] An inverse relation between debt and government consumption is obtained in a non-OLG setting by Schlicht (2006) and Ryoo and Skott (2013).

5 Conclusion

The possibility of a link between public debt and economic growth has received a great deal of attention following the publication of Reinhart and Rogoff (2010) and Kumar and Woo (2010). Assuming that a negative correlation can be found between debt and economic growth, the key question concerns causation.

The analysis in this paper identifies a long-run causal link from growth to debt: a reduction in the long-run rate of growth will tend to produce an increase in the long-run debt ratio. We have also shown that austerity policies can be counterproductive on their own terms (as well as distributionally regressive): reductions in government consumption and/or in entitlement programs for the old increase public debt in the long run.

The model is abstract and focuses on long-run outcomes (steady states). It has other obvious limitations. Most prominently, perhaps, we have assumed a closed economy. Open (and local) economies are in a very different position than sovereign countries that control their own currency; this paper says nothing about the open-economy issues. A second limitation is the neglect of heterogeneity within generations and questions of intra-generational distribution. Public debt may have regressive distributional effects if taxes on wage income are used to finance interest payments to the rich. The possible incentive effects of taxes, third, have been ignored; it should be noted, however, that a higher level of debt need not be associated with higher taxes (section 4). The analysis of these and other complicating issues is beyond the scope of this paper.

Appendix A: Existence and stability of steady states

Existence To see the existence of the solution to (24), let us define

$$f(R) \equiv s\left[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\gamma}} \alpha^{-\frac{1}{1-\gamma}} \sigma^{-\frac{\gamma}{1-\gamma}} R^{\frac{1}{1-\gamma}} - R\right] - 1 - n \tag{A1}$$

where $R = r + \delta$. We then have two cases:

1. $0 \leq \gamma \leq 1$: $f(0) < 0$, $\lim_{R\to\infty} f(R) \to \infty$ and $f(R)$ is continuous. Therefore there exists $R \in (0, \infty)$ such that $f(R) = 0$. The convexity of $f(R)$ ensures the uniqueness.

2. $\gamma < 0$: $f(0) < 0$, $\lim_{R\to\infty} f(R) \to -\infty$ and $f(R)$ is continuous. $f(R)$ is concave and initially increasing but decreasing eventually. Therefore if σ is sufficiently large (for given values of ε and α), there exist two distinct roots, namely, R_1 and R_2 with $R_1 < R_2$. Note that the concavity of $f(R)$ implies

$$f'(R_1) = s\left[\frac{1}{1-\gamma}\left(1+\frac{1+n}{sR_1}\right)-1\right] > 0 \tag{A2}$$

$$f'(R_2) = s\left[\frac{1}{1-\gamma}\left(1+\frac{1+n}{sR_2}\right)-1\right] < 0 \tag{A3}$$

Stability The left-hand and the right-hand side of (23) are strictly decreasing in r_{t+1} and r_t, respectively. Thus, (23) implies that r_{t+1} is strictly increasing in r_t, i.e., $dr_{t+1}/dr_t > 0$ for all r_t. It follows that a fixed point of (23) is locally stable if and only if $dr_{t+1}/dr_t < 1$ at the point.

r_{t+1} is a decreasing function of K_{t+1}/L_{t+1}. To show that $dr_{t+1}/dr_t < 1$ at a stationary point is therefore equivalent to showing that $d(K_{t+1}/K_t)/dr_t$ is positive. Intuitively, at a steady state the capital stock grows at the same rate as the labor force; the steady state is stable if a value of r above the steady growth solution (corresponding to a K/L below the equilibrium) generates a growth rate of the capital that exceeds the growth rate of the labor force. We have

$$\begin{aligned}\frac{K_{t+1}}{K_t} &= s(w_t+\pi_t)\frac{L_t}{K_t} \\ &= s[\left(\frac{\varepsilon}{\varepsilon-1}\right)^{\frac{1}{1-\gamma}} \alpha^{-\frac{1}{1-\gamma}}\sigma^{-\frac{\gamma}{1-\gamma}}(r_t+\delta)^{\frac{1}{1-\gamma}} - (r_t+\delta)] \\ &= f(r_t+\delta)+1+n \end{aligned} \tag{A4}$$

The stability results now follow from the properties of the $f-$function (see above): (i) if $0 \le \gamma \le 1$, the unique stationary solution is stable; (ii) if $\gamma < 0$, the low solution for r is locally stable and the high solution is unstable.

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