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**Abstract**

This paper examines the dynamics of Keynesian models that incorporate feedback effects from the labor market to income distribution, investment, aggregate demand and output. A baseline version of the model can generate endogenous growth cycles, but cumulative divergence and economic collapse also become possible for plausible parameter values. Extensions of the model that include monetary and fiscal policy show greater robustness: the local instability of the stationary point leads to limit cycles (rather than complete collapse), even when large, destabilizing changes are made to parameters describing the private sector. The robustness of the general approach is reinforced by the endogeneity of the fiscal and monetary policy rules.

Key words: growth cycles, Harrodian instability, income distribution, Taylor rule, fiscal policy

JEL numbers: E12, E32, E52, E62

## 1 Introduction

Unlike weather-related fluctuations in agricultural societies, business cycles and recurrent crises in capitalist economies cannot be explained by purely natural causes. In principle, they could arise as a result of shocks operating on an otherwise stable economic system. The technology shocks emphasized by real business cycle theory represents a prominent example, and this approach has been extended in the DSGE literature to include a slew of other shocks. The resulting shock-generated fluctuations are 'exogenous': the economy would converge to a steady growth path in the absence of shocks.

Endogenous business cycles, by contrast, are created by the economic system, even in the absence of external shocks. The analysis of endogenous cycles in capitalist economies goes back to Marx's general law of capitalist accumulation, as described by Marx (1867, chapter 25) and formalized by Goodwin (1967). A different theory of endogenous cycles was developed more than 70 years later, when Keynesian multiplier-accelerator models became mainstream,

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following early contributions from Harrod (1936) and Samuelson (1939): the steady growth path is locally unstable, but nonlinearities in the relations that drive these models can prevent cumulative divergence (Kaldor 1940, Hicks 1950, Goodwin 1951).

The work by Peter Flaschel and his coauthors stands out as a major contribution to the development and refinement of dynamic models in a Keynesian tradition, often augmented by Marx-Goodwin elements that endogenize the distribution of income.[1] Unlike Goodwin's formalization of Marx's general law, which uses a reduced-form real-wage Phillips curve to describe movements in real wages, Flaschel and his coauthors have typically specified separate equations for wage and price inflation to determine the evolution of the wage share, with the explicit equations for inflation also leading naturally to an analysis of monetary policy. This paper adds elements that have been emphasized by Flaschel and his coauthors to the flex-output model in Skott (2015, 2023). Harrodian multiplier-accelerator mechanisms represent the main source of instability, but the model is extended to include destabilizing interactions between income distribution, consumption and aggregate demand ('destabilizing Rose effects', in Flaschel's terminology) as well as economic policy.

The paper makes three main points. First, using empirically based functional forms and parameter values, it is shown that the endogenous cycles generated by the model provide a good fit with observed cyclical patterns for the US economy. Second, the properties of nonlinear dynamic systems can be very sensitive to the precise specification of the system, and it would appear to raise serious questions about the empirical relevance of the model if small changes in parameters can lead to unbounded divergence. These outcomes may be neutralized by strong stabilizing effects of automatic fiscal stabilizers and monetary policy. Economic policy, third, is endogenous, and this endogeneity strengthens the robustness argument: shifts in private sector behavior that threaten to produce cumulative divergence or significant increases in the amplitude of the fluctuations of key macroeconomic variables will almost certainly provoke changes in economic policy.

Section 2 describes and analyzes a stripped-down model of a pure capitalist economy along the lines of Skott (2015). The extended model in section 3 includes wage and price Phillips curves, the influence of expectations and lags on investment, and fiscal and monetary policy. The simulations of the model are presented and discussed in section 4. The concluding section 5 discusses the main results and their robustness.

[1] E.g. Chiarella and Flaschel (2000) and Chiarella et al. (2005). Other recent contributions to the Keynes-Goodwin literature include Barbosa and Taylor (2006) and von Arnim and Barrales (2015).

## 2 A baseline model

### 2.1 Assumptions

Most firms cannot adjust their employment levels instantaneously to meet variations in demand. The hiring process is costly as well as time consuming, and most jobs require some on-the-job training. Like the capital stock, the level of employment is therefore best treated as a state variable in dynamic models of the business cycle.

The net hiring rate reacts to movements in the demand for output, while adjustments in investment aim to keep capital utilization rates at levels that firms deem desirable. If the trajectory of demand were perfectly foreseen, the process would be likely to guide the economy to a steady growth path.[2] In the absence of perfect foresight, however, the dynamics of employment and investment are driven mainly by the behavioral responses to evolving discrepancies between outcomes and expectations. In this setting, the steady growth path need not be stable.

If the productivity of labor and the level of output are predetermined, unanticipated movements in demand lead to discrepancies between output and demand. These discrepancies can show up as quantity rationing, price adjustments or unplanned changes in inventories. Rationing occurs but is generally insignificant in capitalist economies, and inventories are procyclical: at the frequencies that are relevant for business cycles, they tend to amplify the fluctuations, rather than act as buffers against unanticipated changes in demand. This leaves price adjustments and windfall profits as the most plausible manifestation of short-run disequilibrium when output is predetermined. Emphasizing price stickiness, new Keynesians and most post+Keynesians reject this approach.[3] My reading of the evidence suggests that prices are, in fact, much more flexible than commonly believed; Abe and Tonogi (2010), for instance, find that the price of the average item changes roughly every 3 days in a Japanese data set with 3 billion daily observations.[4] In previous work I have therefore analyzed models in which adjustments in prices and profit shares clear the goods market (Skott 1989a, 1989b, Skott and Zipperer 2012).

Building on Skott (2015), this paper takes a different route. Production lags make output predetermined in many sectors of the economy, but there are also businesses without production lags and predetermined levels of output. Haircuts, for instance, are not produced prior to demand, and the consumption of haircuts cannot deviate from the production of haircuts. Similar reasoning applies to many other services. Hair dressers, department stores and restaurants need workers as well as fixed capital. If they have excess capacity of both workers

[2] It is possible to construct models of endogenous cycles in which expectations are always fulfilled (e.g. Grandmont 1985), but the empirical relevance of these models is highly dubious.

[3] The approach is quite Keynesian: accommodating changes in prices and profits were used by Keynes (1930) in his *Treatise on Money*.

[4] Skott (2023, chapter 10) discusses the evidence and the New Keynesian fix-price assumption in greater detail.

and capital, however, their levels of output may become perfectly elastic: the work intensity and labor productivity react to changes in demand.

The model describes a 'flex-output economy' along these lines. There is excess capacity of both labor and capital, and output adjusts instantaneously to the level of demand, within the limits imposed by labor and capital capacity.

**Output and employment** Using a Leontief production function, the capacity constraints are given by

$$Y = yL = \sigma K \leq \min\{y^{\max}L, \sigma^{\max}K\}$$

where $Y, L, K$, $y$ and $\sigma$ denote real output, employment, capital, labor productivity and the output capital ratio. For simplicity, there is no technical change,[5] and the maximum levels of the output-labor and output-capital ratios, $y^{\max}$ and $\sigma^{\max}$, are constant. The Leontief production function implies that the values of $y$ and $\sigma$ are proportional to (and can be used as indicators of) the utilization rates of labor and capital, respectively.[6]

For a number of reasons – including volatility of demand – firms typically aim for some level of 'normal' or 'desired' degree of excess capacity of both labor and capital. The presence of excess capacity allows movements in output to absorb unanticipated demand shocks without any direct rationing, adjustments in prices or unplanned changes in inventories. Employment responds gradually to these movements, expanding when demand is high and the utilization rate of labor exceeds its normal level. The expansion gets blunted, however, if capital constraints become binding for a significant proportion of firms. This happens when the employment capital ratio is high and the average utilization rate of capital approaches its upper limit (if $yL/K$ approaches $\sigma^{\max}$). Low rates of unemployment, moreover, will raise the search and hiring costs and affect the general business climate, thereby putting downward pressure on new hiring.

Formally, the growth rate of employment is specified as a function of signals from the output and input markets (the utilization rate of labor, the employment capital ratio and the employment rate):

$$\hat{L} = h(y, l, e); \qquad h_1 > 0, h_2 < 0, h_3 < 0 \tag{1}$$

where $l = L/K$ and $e = L/N$ denote the employment capital ratio and the employment rate. 'Dots' and 'hats' over a variable will be used throughout the paper to denote the rate of change and proportional growth rate of the variable; i.e., $\hat{L} = \dot{L}/L = (dL/dt)/L$.

**Investment** Capital adjusts more sluggishly than labor, and the accumulation rate is unlikely to be influenced significantly by temporary shocks to demand. The fast-moving utilization rate of labor therefore will have no or only

[5] The analysis would be substantially unchanged in the presence of a constant rate of labor saving technical change. If productivity grows at the rate $g$ and $\tilde{L}$ is employment in hours, the variable $L$ represents effective units of labor, that is, $L = e^{gt}\tilde{L}$.

[6] The rationale for using a Leontief function, rather than specifications that allow siginificant degrees of factor substitution, is discussed in Skott (2023, chapter 9).

limited direct impacts on investment. But investment does react to more sustained changes in demand and, with employment responding to changes in labor utilization, any such changes will be reflected in the employment capital ratio.

These considerations suggest a simple specification with the accumulation rate as an increasing function of the employment-capital ratio:

$$\frac{I}{K} = g_K + \delta = f(l); \qquad f' > 0 \tag{2}$$

where $g_K = \hat{K}$ and $\delta$ are the growth rate of the capital stock and the rate of depreciation. Although this specification of investment may seem unusual, it is closely related to standard investment functions that relate accumulation to the utilization rate of capital. By definition the output capital rate $\sigma$ is equal to the product $yl$. Thus, equation (2) is consistent with the specification in models that define accumulation as a function of $\sigma$ and assume $y = 1$. With labor productivity as a jump variable, however, a simple substitution of $\sigma = yl$ into a standard accumulation function ($I/K = f(\sigma) = f(yl)$) would be unreasonable: firms' investment decisions do not to respond instantaneously to demand shocks that may turn out to be very short-lived. Movements in the labor capital ratio $l$, by contrast, capture longer term changes that can be expected to affect investment.

**Private consumption** Real consumption ($C$) depends on labor income and household wealth,

$$C = c(1-\pi)Y + \nu\Omega \tag{3}$$

The parameters $c$ and $\nu$ are the marginal propensities to consume out of wages and wealth. Household wealth, $\Omega$, is taken to be equal to the capital stock ($K$):[7]

$$\Omega = K \tag{4}$$

Equation (3) is consistent with an old Keynesian consumption function based on the life cycle hypothesis as well as with classical and post-Keynesian theories that emphasize differential saving rates out of profits and wages (see Skott 2023, chapter 3 for a more detailed discussion).

Equations (3)-(4) imply that the ratio of saving to capital can be written

$$\frac{S}{K} = \sigma(1 - c(1-\pi)) - \nu = yl[1 - c(1-\pi)] - \nu \tag{5}$$

**Income distribution** The real wage is taken as constant in the baseline version of the model.[8] The distribution of income need not stay constant, however:

[7] This simplifying assumption excludes important financial complications. Households own fixed capital indirectly through equity shares and other financial assets, and the ratio of financial valuation to the replacement cost of capital (Tobin's q) varies over time.

[8] With labor saving technical change, it would be the real wage per effective unit of labor that were constant, the real wage per hour growing at the same rate as technical change.

cyclical movements in labor productivity will affect the profit share, which is definitionally related to the real wage $\omega$ and the productivity of labor,

$$\pi = 1 - \frac{\omega}{y} \tag{6}$$

**Equilibrium conditions** The equilibrium condition for the goods market requires that

$$I = S \tag{7}$$

## 2.2 Implications

Equations (2) and (5)-(7) determine the utilization rate of labor ($y$) as a function of the employment capital ratio ($l$):

$$yl[1 - c(1-\pi)] = [g_K + \delta + \nu] \tag{8}$$

or

$$y = \frac{f(l) + \nu}{l} + c\omega \tag{9}$$

The dynamics of the economic system now follow from equations (1)-(2) and (9). If the labor force grows at a constant rate, $n$, the model produces a two-dimensional system of differential equations in the state variables $e$ and and $l$:

$$\dot{l} = l[h(y(l), l, e) - f(l) + \delta] \tag{10}$$

$$\dot{e} = e[h(y(l), l, e) - n] \tag{11}$$

The system (10)-(11) has at most one non-trivial stationary solution with $l > 0, e > 0$. To see this, note that if $\hat{e} = 0$, then $h(y(l), l, e) = n$. The stationarity of both $e$ and $l$ therefore requires that $f(l) = \delta + n$. The accumulation function $f$ is monotonically increasing, and this equation can have at most one solution, $l^*$. A meaningful solution exists if the net accumulation rate falls below the natural growth rate for a utilization rate of zero but rises above the natural rate for sufficiently high utilization rates; formally, it is required that $f(0) < \delta + n$ and that $f(l) > \delta + n$ for $l$ values above some value. A capitalist economy would not be viable if these behaviorally plausible conditions on accumulation failed to be satisfied. The conditions are not sufficient, however.

The function $h(y(l), l, e)$ is monotonically decreasing in $e$ and, having pinned down $l^*$, equation (11) can be used to determine $e^*$ : a solution with $0 < e \leq 1$ exists if $h(y(l^*), l^*, 0) > n$ and $h(y(l^*), l^*, 1) \leq n$. The second of these conditions must hold, almost by definition: employment cannot increase faster than the labor force if the economy is at full employment. For present purposes, I shall assume that the first condition is also satisfied. If the condition fails to be met, the economy will experience secular stagnation: aggregate demand will be insufficient (in the absence of government intervention) to support a steady growth path with a growth rate that equals the natural growth rate $n$; Skott (2023, chapter 11) discusses secular stagnation and functional finance.

Assuming the existence of a stationary solution, the local stability properties are determined by the trace and determinant of the Jacobian matrix (evaluated at the stationary point). The Jacobian is given by

$$J(l,e) = \begin{pmatrix} l^*[h_1 y' + h_2 - f\ '] & l^* h_3 \\ e^*[h_1 y' + h_2] & e^* h_3 \end{pmatrix}$$

The trace and determinant are

$$\begin{aligned} Det(J) &= -l^* f\ ' e^* h_3 \\ Tr(J) &= l^*[h_1 y' + h_2 - f\ '] + e^* h_3 \end{aligned}$$

Local stability requires a positive determinant and a negative trace. The determinant is unambiguously positive, but the trace, which contains both positive and negative terms, cannot be signed without additional assumptions about relative magnitudes. A case can be made that the trace will be positive and the stationary solution locally unstable, but stability cannot be excluded; see Appendix A.

## 2.3 Simulations

The simulation in figure 1a illustrates a case with local instability. The natural growth rate, the depreciation rate and the real wage are set at $n = 0.03, \delta = 0.07$ and $\omega = 0.7$. The consumption parameters are $c = 1, \nu = 0.05$, yielding a saving capital ratio of

$$\frac{S}{K} = \pi y l - 0.05 \tag{12}$$

The investment function is taken to be linear; by definition, however, gross investment cannot be negative. Thus,

$$\begin{aligned} \frac{I}{K} &= g_K + \delta = f(l) = \max\{0, n + \delta + (l - 0.5)\} \\ &= \max\{0, 0.1 + (l - 0.5)\} \end{aligned} \tag{13}$$

The employment expansion function (1), by contrast, is likely to be highly non-linear. Managerial resources "create a fundamental and inescapable limit to the amount of expansion a firm can undertake at any time" (Penrose 1959, p. 48), and very rapid contractions of the labor force may also impose high costs as morale and productivity suffer.[9] Additional nonlinearities are associated with the employment rate and the employment capital ratio. It matters little for the business climate and the ability of firms to find and hire workers with the required skills whether the employment rate is 50 percent or 51 percent;

[9] Monthly quit rates of non-farm workers in the US of 1.2% in the 2009 recession and 2.4% at the pre-COVID peak indicate that many smaller contractions of emplyoment can be achieved without the costs associated with layoffs.

an increase from 95 to 96 percent, by contrast, could have a large impact. Analogously, small changes in the employment capital ratio have no significant effect on the prevalence of capital constraints if the initial value of $l$ is low. Thus, the sensitivity of employment growth to changes in the aggregate employment rate and the employment capital ratio will be weak at low levels of $e$ and $l$ but strengthen with increases in the levels of employment or capacity utilization. These nonlinearities are captured by the following specification of the growth rate employment:

$$\begin{aligned}\hat{L} &= h(y, l, e) \\ &= \frac{0.3}{1 + \exp(-20(y - 1 + 0.02(0.1^{-0.5} - (1-e)^{-0.5})) + 0.01((0.1^{-0.5} - (0.6-l)^{-0.5})))} \\ &\quad -0.12 \end{aligned} \tag{14}$$

The functional forms and parameter values of the investment and employment expansion functions draw on the specifications and econometric estimates in Skott and Zipperer (2012). Their regression results imply that investment reacts strongly to sustained changes in demand and that the growth rate of output responds positively to the profit share (which is related to the utilization rate of labor) and negatively to the employment rate and capital utilization. The present model differs from the one in Skott and Zipperer (2012) with respect to price and output flexibility, but the parameters are in line with their estimates.

Figure 1a about here

With these functional forms and parameters, the model has a stationary solution at $y^* = 1, l^* = 0.5, e^* = 0.9, \pi^* = 0.3$. Figure 1a depicts the outcome over 500 periods with the initial values set to $l(0) = 0.52, e(0) = 0.91$ The stationary solution is locally unstable, and the model produces convergence to a limit cycle with clockwise cycles in the $(e, \sigma), (e, \pi), (e, \hat{Y}), (\pi, \hat{Y})$ and $(\sigma, \hat{Y})$ spaces. These movements are consistent with observed patterns in the US economy (Zipperer and Skott 2011).

Inevitably, highly stylized models like (10)-(11) also fail to capture many properties of real-world cycles; the calibrated model, for instance, produces a static, reduced-form functional relation between the profit share and the output capital ratio, which precludes the clockwise cycles in the $(\sigma, \pi)$ space seen in the data. More importantly, like most simulations of non-linear dynamic systems, the results are sensitive to changes in parameter values and functional forms. The functional forms and parameters are, I would argue, behaviorally and empirically plausible, but other plausible parameters and functional forms can produce very different results.

The fragility of the results can be illustrated by changing the parameter in the investment function. If the coefficient on $l$ is reduced slightly from 1 to 0.985, the limit cycle disappears and the stationary solution becomes locally stable; increasing the coefficient above 1.075, on the other hand, generates unbounded fluctuations and, eventually, a complete collapse with the employment rate converging to zero.

The window of cycles is not quite as narrow as these qualitative results suggest. For practical purposes it makes little difference whether the system produces slow oscillatory convergence to the stationary solution or convergence to a closed orbit. But the point remains: the simulation results are fragile, and I deliberately chose a sensitivity of investment to changes in utilization at the low end of the long-run estimates in Skott and Zipperer (2012) in order to illustrate the possibility of limit cycles. Figures 1b and 1c depict a stable case with an investment parameter of 0.97 and a case of cumulative divergence with an investment parameter of 1.08, respectively.

Figures 1b and 1c about here

The fragility also shows up if the real wage is endogenized. Using a linear real-wage function, let

$$\omega = 0.7 + a(e - 0.9) \tag{15}$$

The real wage now responds to deviations from the stationary value of $e$ but, keeping the functional forms and parameter values of the original simulation, the stationary solution is unchanged. The sensitivity of the outcome to changes in the parameter $a$, can be illustrated by noting that (retaining all other parameters in the simulation) the economy heads for collapse with $e \to 0$ for any $a$ value above 0.033; parameter values below -0.01, by contrast, make the stationary point stable.

Following Flaschel and his coauthors, the real wage can also, more realistically, follow a dynamic 'real-wage Phillips curve',[10]

$$\hat{\omega} = \alpha_1(e - e^*) - \alpha_2(\sigma - \sigma^*) \tag{16}$$

In this case positive coefficients on $(e - e^*)$ are highly destabilizing, while negative coefficients on $(\sigma - \sigma^*)$ are stabilizing. Figures 2a-2b illustrate the patterns for $(e, \sigma)$. Figure 2a, which uses the parameters $\alpha_1 = 0.01$ and $\alpha_2 = 0$, shows how even a tiny employment effect on real wage growth is sufficient to produce divergence. In figure 2b with $\alpha_1 = 0$ and $\alpha_2 = 0.1$, the stabilizing negative feedback from capital utilization to real-wage growth produces convergence to the steady growth path.[11] In both cases, the investment, consumption and employment expansion functions as well as the initial values are as in figure 1a.

Figures 2a and 2b about here

[10] The cyclical pattern of real wages is not entirely clear, but many studies suggest that the level or growth rate of real wages depends positively on the employment rate. Solon et al. (1995) find a positive relation in levels; Flaschel and Krolzig (2006) a positive relation for the growth rate; Abraham and Haltiwanger (1995) suggest that the evidence is inconclusive.

[11] For values of $\alpha_1$ below 0.1 there is convergence to a limit cycle with progressively smaller amplitude as $\alpha_2$ increases.

# 3 An extended model with monetary and fiscal policy

The complete absence in the baseline model of a public sector and economic policy misrepresents modern capitalist economies. Central banks routinely adjust interest rates in response to economic performance, automatic stabilizers are significant, even in capitalist economies with relatively small public sectors, and discretionary fiscal policy can be and has been used, especially when monetary policy and automatic stabilizers prove inadequate to prevent major recessions. Furthermore, the sensitivity of the baseline model to cyclical movements in the real wage make it questionable to freeze the real wage by assumption, and the specification of investment also has serious weaknesses.

The extensions in this section address these problems at the cost of increasing the dimensionality of the dynamic system.[12] Harrodian multiplier-accelerator mechanisms are still the main source of instability, with feedback effects from the labor market playing a key role in turning local instability into bounded fluctuations. But the Harrodian mechanism becomes more complex; endogenous movements in real wages and inflation contribute to the dynamics; important stabilizing feedback effects from the labor market are mediated by economic policy and automatic fiscal stabilizers.

## 3.1 Extensions

**Fiscal and monetary policy** Government spending on goods and services is relatively stable, a feature that can be captured in a stylized way by linking spending to potential output. There are two natural indicators of potential output: the level of output associated with a normal utilization of the capital stock ($\sigma^* K$) and the level of output associated with having the employment rate and the productivity of labor at their steady growth values ($y^* e^* N$). A simple linear specification along these lines assumes that[13]

$$G = \gamma_1 \sigma^* K + \gamma_2 y^* e^* N$$

where $N$ is the labor force and $e^*, y^*$ and $\sigma^*$ denote the employment rate, labor productivity and output capital ratio along the steady growth path.

Leaving out discretionary fiscal responses, however, would underestimate the influence of fiscal stabilization, especially in states with high unemployment and interest rates at the zero lower bound. The US fiscal stimulus in 2009 and again, more dramatically, during the COVID pandemic provide recent examples. Thus, I shall add a third, cubic term: this term is relatively unimportant when the

[12] There is no attempt to include open economy issues, which limits to applicability of the model. Arguably, however, the US economy can be treated as approximately closed, at least between 1950 and the 1980s. Thus, empirical assessments of the model should focus on its ability to match the stylized patterns of the US, rather than those of smaller and more open economies.

[13] This specification is similar to one used by Thompson (2018). Fazzari et al. (2013) also emphasize the stabilizing effects of government spending in a related model.

employment rate is close to its steady growth value; positive when $e$ is below the steady growth value and increasing rapidly as $e$ declines; negative when the economy is overheating and decreasing rapidly as employment rises significantly above the steady growth path.[14]

High debt ratios, finally, often generate downward pressures on government spending. The strength of this effect is not obvious, but the 'sustainability of government debt' has been a constant theme in economic policy, and it would seem clear that there is some negative feedback from debt levels to government spending.

Formally, let

$$G = \gamma_1 \sigma^* K + \gamma_2 y^* e^* N + \gamma_3 (e^* - e)^3 K - \gamma_4 \frac{B}{p} \tag{17}$$

where $B$ is nominal government debt. The extended model includes an endogenous determination of inflation. Thus, the price level cannot be normalized to one, and nominal debt must deflated by the price $p$; unlike $B$, the variables $G$ and $K$ are in real terms.

Tax revenues may be determined largely by current income, but many transfers depend on the employment rate; unemployment benefits represent a prime example. As a simple specification, assume that net taxes (in real terms) are given by[15]

$$T = t_1 Y - t_2 (e^* - e) y^* N \tag{18}$$

The dynamics of nominal government debt follows the standard equation

$$\dot{B} = pG - pT + rB \tag{19}$$

where $r$ is the nominal rate of interest on government debt. The tax, transfer and spending parameters will be calibrated to ensure that a balanced government budget and zero debt become consistent with steady growth.[16] Deficits (surpluses) will appear in recessions (booms), however, leading to fluctuations in public debt; because of asymmetries between expansions and downturns, the average level of debt over the cycle will not in general be equal to zero. The

[14] Almost by definition, discretionary policy will be lumpy and irregular; it will not follow a smooth function. The cubic specification should be seen as a simple, mechanical way of approximating these less regular responses and the increasing likelihood of discretionary intervention when the employment rate deviates significantly from its steady growth value.

[15] Interest payments on government debt are typically taxed, and tax revenues could be specified as

$$T = t_1 (Y + r\frac{B}{p}) - t_2 (e^* - e) y^* N$$

Simulation results with this alternative specification are very similar to those reported in section 4.

[16] Skott (2023, chapter 11) considers fiscal policy and the role of public debt in the long run if weak demand threatens to produce secular stagnation (cf. above, p. 6).

trajectory of the debt capital ratio ($b = B/(pK)$) is determined by

$$\begin{aligned}
\dot{b} &= b[\hat{B} - \hat{p} - \hat{K}) = b[\frac{pG - pT + rB}{B} - \hat{p} - g_K] \\
&= \frac{pG - pT + rB}{pK} - (\hat{p} + g_K)b \\
&= \frac{G}{K} - \frac{T}{K} + (r - (\hat{p} + g_K))b \qquad (20)
\end{aligned}$$

Taylor rules for monetary policy specify the target interest rate as an increasing function of inflation and the deviation of output from potential output. The evidence shows significant inertia, however, with the adjustments happening gradually (Clarida et al. 2000). Formally, let

$$\dot{r}^d = \lambda_r(r^T - r^d) \qquad (21)$$

$$r^T = \bar{r} + \theta_1(\hat{p} - \hat{p}^T) + \theta_2(e - e^*) + \theta_3(\sigma - \sigma^*) \qquad (22)$$

The central bank would like to change its interest rate $r^d$ in proportion to the difference between the target rate $r^T$ and the current value of $r^d$, with the parameter $\lambda_r$ indicating the speed of adjustment. The trajectory of interest rates is subject to a ZLB-constraint, however: the actual nominal rate is constrained to be positive,[17]

$$r = \max\{0, r^d\} \qquad (23)$$

**Wages, prices and the profit share** Using specifications that follow the analysis and empirical results in Flaschel and Krolzig (2006) and Diallo et al. (2011), it is assumed that the rates of wage and price inflation are determined by separate, expectations augmented Phillips curves. Wage inflation is a function of the employment rate, the utilization rate of capital, the profit share and expected price inflation, while price inflation depends on the utilization rate of capital, wage inflation, the profit share and the expected inflation rate:[18]

$$\hat{w} = \beta_e(e - e^*) + \beta_\sigma(\sigma - \sigma^*) + \beta_\pi(\pi - \pi^*) + x \qquad (24)$$

$$\hat{p} = \beta_1(\sigma - \sigma^*) + \beta_2\hat{w} - \beta_3(\pi - \pi^*) + (1 - \beta_2)x \qquad (25)$$

where $w, p$ and $x$ are the nominal wage, price and expected price inflation. Expected inflation follows an adaptive process

$$\dot{x} = \lambda_x(\hat{p} - x) \qquad (26)$$

[17] The policy interest rate has been negative in some economies, including the EU. It makes no substantial difference, however, whether the lower bound is at $0, -0.5\%$ or $-1\%$. Following standard practice, I shall assume that the zero lower bound is in fact zero.

[18] Diallo et al. (2011) also include endogenous variations in the utilization of labor. Unlike in the present model, however, they assume perfect flexibility with respect to the number of working hours. The productivity per hour is taken to be constant, with changes in hours affecting total wage income rather than the profit share.

Using equations (24) and (25), the inflation rate and the dynamics of the real wage can be written as

$$\begin{aligned}\hat{p} &= \beta_2\beta_e(e-e^*)+(\beta_1+\beta_2\beta_\sigma)(\sigma-\sigma^*)+(\beta_2\beta_\pi-\beta_3)(\pi-\pi^*)+x \\ &= \alpha_{p1}(e-e^*)+\alpha_{p2}(\sigma-\sigma^*)+\alpha_{p3}(\pi-\pi^*)+x \qquad (27)\\ \hat{\omega} &= (1-\beta_2)\beta_e(e-e^*)-[\beta_1-(1-\beta_2)\beta_\sigma](\sigma-\sigma^*)+[(1-\beta_2)\beta_\pi+\beta_3](\pi-\pi^*) \\ &= \alpha_{\omega 1}(e-e^*)-\alpha_{\omega 2}(\sigma-\sigma^*)+\alpha_{\omega 3}(\pi-\pi^*) \qquad (28)\end{aligned}$$

where $\alpha_{p1}=\beta_2\beta_e, \alpha_{p2}=\beta_1+\beta_2\beta_\sigma, \alpha_{p3}=\beta_2\beta_\pi-\beta_3, \alpha_{\omega 1}=(1-\beta_2)\beta_e, \alpha_{\omega 2}=\beta_1-(1-\beta_2)\beta_\sigma, \alpha_{\omega 3}=(1-\beta_2)\beta_\pi+\beta_3$. Phillips curves often leave out the utilization rate of capital. Both employment and utilization influence potential output, however, which supports specifications like equation (27) that include both variables.[19] Equation (28) implies that the real wage can be 'labor market led' or 'goods market led', in the terminology of Diallo et al. (2011): depending on parameter values and on the reduced-form correlation between $e$ and $\sigma$, the growth of real wages may be positively or negatively related to the employment rate.

Equations (24)-(27) overestimate the downward flexibility of nominal wages and prices, which even in the depth of the great depression did not fall at an annual rate below -10%.[20] The imposition of a lower limit on the inflation rate approximates this stickiness in a simple way:

$$\hat{p}=\max\{\alpha_{p1}(e-e^*)+\alpha_{p2}(\sigma-\sigma^*)+\alpha_{p3}(\pi-\pi^*)+x, \hat{p}_{\min}\} \qquad (29)$$

**Investment** Investment plays a central role in all Keynesian theory – especially so in dynamic models with Harrodian instability – and the baseline investment function in equation (2) leaves out potentially important complications. First, investment requires financing, and an increase in the costs of finance (in the real rate of interest) is likely to exert a dampening influence. Second, an inability of some firms to obtain external finance (or, more generally, their inability to obtain it on reasonable terms) implies that the profit share may have a direct positive influence on investment (e.g. Fazzari et al. 1988), an influence that is reinforced by a more general effect of the profit share on animal spirits. Third, there are likely to be feedback effects from the labor market to accumulation: the size of the reserve army of the unemployed impacts the general business climate and the willingness of firms to invest.

The accumulation rate, fourth, will be affected by expected future growth rates of demand. If firms were identical and the economy fluctuated around a steady growth path, the expected medium-term growth rate could be taken as constant: the expectations could become anchored to the long-run steady growth rate. But firms differ. Some firms experience high growth rates while

[19] The existence of a well-defined Phillips curve can be questioned. For present purposes, however, I set aside these issues; see Skott (2023, chapters 5-6 for a broader discussion of Phillips curves and wage setting.

[20] Akerlof et al (1995) and Shafir et al. (1997) discuss 'money illusion' and downward stickiness.

others stagnate or decline. Hence, even if aggregate output fluctuates around a steady growth path with a constant employment rate, the growth expectations of individual firms will not be anchored to the steady growth rate. When observing an increase in demand, each firm has to disentangle the role of temporary shifts (which can be firm specific or related to aggregate demand) from sustained shifts in demand for its output. Weighing these possibilities, firms are likely to adjust their medium-term demand expectations partially in response to movements in aggregate demand.[21]

Fifth, investment is subject to both decision and implementation lags. The specification of investment as a function of the employment capital ratio (rather than the output capital ratio) excludes an immediate impact of demand shocks. The influence comes gradually as employment responds to changes in the utilization rate of labor. This indirect introduction of a delay between changes in demand and investment goes some way towards capturing the decision lags. But some investment projects – the construction of major new plants, for instance – involve significant lags between the decision to invest and the start of actual investment as well as prolonged periods of ongoing investment. To approximate these lags between decision and implementation in a continuous-time setting, a distinction can be made between actual and target investment, with actual accumulation rates adjusting towards the target rate. This gradual adjustment of investment is analogous to the capital adjustment principle, but now applied to investment levels rather than capital stocks.

Formally, adding the real interest rate $(r - x)$, the profit share, the employment rate and the expected, medium-term growth rate $(\rho)$ to the target investment function and introducing an implementation lag, let

$$(\frac{I}{K})^T = \max\{0, \rho + f(l, r - x, \pi, e)\}; \; f_1 > 0, f_2 < 0, f_3 > 0, f_4 < 0 \quad (30)$$

$$\dot{\rho} = \lambda_\rho(\hat{L} - \rho); \quad \lambda_\rho > 0 \quad (31)$$

$$\dot{g}_K = \frac{d}{dt}(\frac{I}{K} - \delta) = \lambda_{g_K}((\frac{I}{K})^T - \frac{I}{K}); \quad \lambda_{g_K} > 0 \quad (32)$$

where $(I/K)^T$ denotes the target rate of gross accumulation. As in section 2, gross investment is subject to a non-negativity constraint.

**Private consumption and saving** Consumption $(C)$ now depends on disposable labor income, employment-dependent transfers (which are treated like wage income) and household wealth,

$$C = c[(1 - \pi)(1 - t_1)Y + t_2(e^* - e)N] + \nu\Omega \quad (33)$$

[21] The basic argument is analogous to that in Phelps (1969) and Lucas (1972). Unlike in Phelps and Lucas, however, there is no presumption of rational expectations, and the focus is on firms and their assessment of whether demand shifts are permanent or temporary, rather than on household-producers' assessment of whether a price increase reflects a rise in the relative price of their output.

### 3.2 Short run equilibrium and dynamic system

The equilibrium condition for the goods market includes government spending and taxation,

$$Y = C + I + G \tag{34}$$

Using equations (6), (17)-(18) and (34), equation (34) determines the utilization rate of labor as an implicit function of $l, g_K, e, \omega$ and $b$:

$$y = \frac{g_K + \delta + \gamma_1\sigma^* + \nu(1+b) + \gamma_3(e^* - e)^3 - \gamma_4 b}{l}] + \gamma_2 y^* \frac{e^*}{e} + c(1-t_1)\omega + ct_2 y^*(\frac{e^*}{e} - 1) \tag{35}$$

The extended model generates the following system of differential equations:

$$\begin{aligned}
\dot{l} &= l(\hat{L} - g_K) = l[h(y,l,e) - g_K] \\
\dot{g}_K &= \lambda_{g_K}((\frac{I}{K})^T - \frac{I}{K}) = \lambda_{g_K}(\rho + f(l, r - x, \pi, e) - (g_K + \delta)) \\
\dot{\rho} &= \lambda_\rho(\hat{L} - \rho) = \lambda_\rho[h(y,l,e) - \rho] \\
\dot{e} &= e(\hat{L} - n) = e[h(y,l,e) - n] \\
\dot{x} &= \lambda_x(\hat{p} - x) \\
\dot{\omega} &= \omega[\alpha_{\omega 1}(e - e^*) - \alpha_{\omega 2}(\sigma - \sigma^*) + \alpha_{\omega 3}(\pi - \pi^*)] \\
\dot{b} &= \frac{G}{K} - \frac{T}{K} + (r - \hat{p} - g_K)b \\
\dot{r}^d &= \lambda_r(r^d - r)
\end{aligned}$$

There are eight state variables $(l, g_K, \rho, e, x, \omega, b, r^d)$, with $\pi, r, \hat{p}, G/K$, $T/K$ and $y$ determined statically as functions of these state variables (equations (6), (17)-(18), (22), (29) and (35)).

Even basic properties of the system are indeterminate without additional assumptions. Depending on functional forms and parameters, for instance, the system may have no stationary solution or it could have multiple solutions. Fortunately, we have relatively good information about some of the relations. The investment function is the main exception, but even in this case, ballpark values of the parameters can be identified.

The next section uses empirically based calibrations and simulations to illustrate important properties of the system. As in section 2, variations in the sensitivity of investment to changes in the employment capital ratio will be used to examine the robustness of the model.

## 4 Simulations

All simulations retain the employment growth function $h(y, l, e)$ in equation (14) with respect to both functional form and parameter values; the argument for this specification, which was outlined in section 2, still applies. The investment function is also kept linear, as in section 2, but variables have been added and the specification now applies to target investment:

$$(\frac{I}{K})^T = \delta + \rho + \rho_1(l - 0.5) + \rho_2(\pi - 0.375) - \rho_3[(r - x) - 0.03] - \rho_4(e - 0.9)$$

## 4.1 Simulations with given policy rules

The simulations in figure 3 use the parameter values in table 1. The parameter values in the Phillips curve and the equation for real-wage dynamics are in line with the specifications and econometric results in Flaschel and Krolzig (2005) and Diallo et al. (2011). The two papers use slightly different specifications. The values of $\alpha_{p1}, \alpha_{p2,}\alpha_{p3,}\alpha_{w1}$ and $\alpha_{w2}$ correspond to the findings in Flaschel and Krolzig (with rounding and adjustments for differences in definitions as well as conversion of the quarterly estimates to a continuous time setting). The feedback from the profit share to wage and price inflation makes economic sense and its inclusion (a positive $\alpha_{w3}$) finds support in Diallo et al. Neither Flaschel and Krolzig nor Diallo et al. estimate the expectation process ($\lambda_x$). Instead, it is assumed that expected inflation is equal to an unweighted moving average over the previous 12 quarters (Flaschel and Krolzig) or a weighted 12 quarter moving average with linearly declining weights (Diallo et al.). These assumptions provide an average lag that is comparable to that obtained by the autoregressive formulation with $\lambda_x = 0.3$. This speed of adjustment of inflation expectations is also consistent with Blanchard's (2016) findings.

**Table 1: Parameter values**

| | |
|---|---|
| Phillips curve | $\alpha_{p1} = 0.3, \alpha_{p2} = 0.4, \alpha_{p3} = 0, \hat{p}_{\min} = -0.1$ |
| Inflation expectations | $\lambda_x = 0.3$ |
| Real wage dynamics | $\alpha_{\omega 1} = 0.5, \alpha_{\omega 2} = 0.3, \alpha_{\omega 3} = 0.5$ |
| Target investment | $\rho_1 = 0.89, \rho_2 = 0.2, \rho_3 = 0.2, \rho_3 = 0.1, \rho_4 = 0.2, \delta = 0.07$ |
| Investment adjustment | $\lambda_\rho = 0.4, \lambda_{g_K} = 2$ |
| Government consumption | $\gamma_1 = 0.1, \gamma_2 = 0.1, \gamma_3 = 75, \gamma_4 = 0.05$ |
| Taxation | $t_1 = 0.2, t_2 = 0.04$ |
| Taylor rule | $\lambda_r = 2, \theta_1 = 2, \theta_2 = 0.5, \theta_3 = 0.5, \bar{r} = 0.05, \hat{p}^T = 0.02$ |
| Natural growth rate | $n = 0.03$ |
| Private consumption | $c = 1, \nu = 0.05$ |

The government spending and tax parameters imply that the government budget is balanced along a steady growth path if there is no initial debt. The share of government consumption in GDP is 0.2, below the share in the US from 1951-1992 and above the average for 1992-2021. The parameter describing employment dependent transfers is set to 0.04. The average unemployment replacement ratio in the US is about 0.4, but only a fraction of the unemployed, ranging between 9 percent in Mississippi and 55 percent in Massachusetts, get unemployment benefits. Relatively low paid workers experience greater employment volatility, and a one percent increase in unemployment may only reduce aggregate wage income by about one third of a percent of GDP. Assuming that about 30 percent of the unemployed receive benefits at an average replacement ratio of 0.4, the direct fiscal effect of increased unemployment makes $\tau_2$ equal to 0.04. This calculation leaves out other sources of increased transfers, including food stamps and medicaid; thus, the parameter value may be on the low side. The parameter $\gamma_3$ describing discretionary policy is calibrated based on the US stimulus package in 2009, when a fall in the employment rate of about 5 percentage points was met with a discretionary fiscal stimulus of about 5 percent of GDP spread over about two years.[22] The value of $\gamma_4$ implies that a 10 percentage point increase in the debt to GDP ratio reduces the share of discretionary in GDP spending by half a percentage point.

The Taylor rule and its parameter values follow the findings in Clarida et al. (2000) and Diallo et al. (2011). The natural growth rate ($n$), the consumption parameters ($c, \nu$) and the depreciation rate ($\delta$) match standard assumptions.

There is greater uncertainty with respect to the parameters of the investment function. My reading of the available evidence suggests large positive long-run effects of utilization, some positive effect of the profit share and some negative effects of the employment rate and real interest rates.[23] Including the interest rate is standard, but the cost of finance may have limited effects on investment in recessions. The relevant risk adjusted rate on corporate loans and bonds, moreover, does not always move with the policy rate; the spread between the policy rate and the risk adjusted interest rate on business finance tends to increase in recessions. As an extreme example, the policy rate – the federal funds rate – was reduced by 5 percentage points between July 2007 and the end of 2008. Yet, during that same period the yield on corporate Baa bonds increased by more then 2 percentage points.[24] The investment parameters reflect this evidence and represent, I would argue, reasonable ballpark estimates.

The functional forms and parameter values imply that the model has a stationary solution $E_1$ with $l^* = 0.5, e^* = 0.9, y^* = 1, g_K^* = 0.03, x^* = 0.02, \gamma^* = 0.03, r^* = 0.05, \omega^* = 0.625, b^* = 0$. Simulations with a range of different initial values indicate that, given all other parameters in table 1, this stationary solution is locally stable if the investment parameter $\rho_1$ is below 0.89 and locally unstable for values above 0.89. The convergence is oscillatory in the stable case with $\rho_1$ below 0.89 and quite slow for $\rho_1$ values above 0.5. In the unstable case,

[22] The calibration implies that a fall in $e$ of 0.055 will generate a discretionary fiscal package of $0.0125K$ or, with a capital output ratio of 2, a discretionary 2.5 percent rise in $G/Y$.

The discretionary fiscal response to rising unemployment during the COVID pandemic was stronger than suggested by the calibration, but the COVID response may be seen as a special case.

[23] Skott and Zipperer (2012) present econometric evidence for the US using a related specification. The mainstream literature on investment is sparse and mainly focused on specifications that are quite different, often building on Tobin's q theory with convex adjustment costs. Post-Keynesian studies have also examined the investment function empirically. The dominant Kaleckian strand of this literature has largely taken it for granted that the effect of utilization rates on investment is small; econometric studies that purport to support the Kaleckian approach sometimes show otherwise, however (see Skott 2012).

[24] Changes in interest rates could impact consumption as a result of valuation effects on financial wealth. However, the model takes household wealth as equal to $pK + B$, which precludes this possibility.

the trajectories remain bounded, converging to a limit cycle even for values of $\rho_1$ that are far above any plausible range. The amplitude of the asymptotic cycle depends on $\rho_1$: it is zero for $\rho_1 = 0.89$, increases gradually with $\rho_1$, and for $\rho_1 = 10$ the convergence is to a limit cycle with employment fluctuating between 0.79 and 0.98.[25]

Figure 3 about here

Figure 3 shows bivariate patterns for $(e, \sigma), (e, \pi), (e, \hat{p}), (e, r)$ and $(e,\omega)$ from a simulation with the $\rho_1$ parameter at the borderline value ($\rho_1 = 0.89$). The initial values of the employment rate, the employment capital ratio and the debt ratio are $e_0 = 0.92$ and $l_0 = 0.51$, with all other state variables equal to their values at the stationary solution $E_1$; the figure depicts patterns for the first 100 periods. The orientations of the cycles match the regular patterns of all bivariate fluctuations identified by Zipperer and Skott (2011) as well as the 'labor-market led' movements in real wages emphasized by Diallo et al. (2011).[26] The relative amplitudes of the key variables are roughly correct, but with the implied amplitude of the capital utilization rate on the low side, relative to the amplitudes of the profit share and the employment rate. With $\rho_1$ at the threshold of instability, the stationary solution is stable or, equivalently, there is convergence to a limit cycle with zero amplitude. But the convergence is slow and oscillatory, with a cycle length of about 10 years.

Figure 4 about here

A variation of the simulation removes the feedback from debt to government spending (setting $\gamma_4 = 0$). If the initial value of the debt ratio is positive, the system now produces divergence, even if positive debt is the only deviation of the initial position from the stationary solution: the debt ratio rises without limit;

[25] Using the stationary values associated with $E_1$ in the equations for government consumption, taxes, real wages and inflation, the system has a second non-trivial stationary solution ($E_2$). The interest and inflation rates are at their lower bounds at this solution, while the values of the other state variables depend on the parameters. When $\rho_1 = 0.89$, this solution has $e^{**} = 0.865, l^{**} = 0.494, \sigma^{**} = 0.489, y^{**} = 0.990, \pi^{**} = 0.404, \rho^{**} = 0.03, x^{**} = -0.1, g_K^{**} = 0.03, r^{d**} = 0, \omega^{**} = 0.590, b^{**} = -0.387$.

The stationary solution at $E_2$ is unstable. In the case with $\gamma_4 = \nu$, however, the value of the debt ratio $b$ has no influence on the other state variables, and the trajectories of $l, g_K, \rho, e, x, \omega, r^d$ may converge to the stationary solution associated with $E_2$. The basin of attraction for this outcome is small: it appears, for instance, that if $\rho_1 = 0.89$, all trajectories converge to the 'good solution' $E_1$ as long as the initial value of expected inflation exceeds $-0.03$. $E_2$, moreover, is a saddlepoint and only by a fluke will trajectories that start within this basin of attraction that also generate convergence of $b$ to $b^{**}$; the debt ratio will almost certainly exhibit cumulative divergence. This divergence would generate unbounded – and thereby unsustainable – movements in $C$ and $G$, with the $C + G$ staying constant.

[26] The baseline model exhibited 'labor market led' cycles in the wage share (clockwise cycles in $(e, \pi)$ but assumed a constant real wage.

Zipperer and Skott document clockwise cycles in $(e, \sigma), (e, \pi), (\sigma, \pi), (e, \hat{Y}), (\sigma, \hat{Y})$ and $(\pi, \hat{Y})$. They also examined the bivariate patterns for $(e, \hat{K}), (\sigma, \hat{K})$ and $(\pi, \hat{K})$. These patterns are less clear, however.

see figure 4, which uses $b_0 = 0.03$.[27] Intuitively, debt has an expansionary effect on consumption (which is not being offset by lower government spending when $\gamma_4 = 0$), and monetary policy makers react to the expansion by increasing the rate of interest, which raises the growth rate of government debt. As the debt ratio rises, the expansionary effect on rising debt on private consumption comes to dominate the contractionary effects of rising interest rates on investment, generating an explosive path of rising $e, \sigma.\pi, y, \omega, r, x$ and $b$.[28]

A negative initial debt ratio, conversely, can generate explosive growth in private sector debt and downward divergence with private consumption going to zero. This outcome requires that the nominal interest rate hits the ZLB, leading to progressively falling inflation and increasing real rates of interest.[29]

It may be noted that an increase in the minimum inflation rate tends to dampen the explosiveness of the movements in the debt ratio. This dampening effect is intuitively obvious in the case with negative public debt, where the destabilizing rise in real interest rates happens because interest rates are non-negative and inflation rates fall below zero. It also applies when debt is positive, however: the floor under inflation tends to reduce the real interest rate in downturns, thereby dampening or eliminating the trend rate of growth in the debt ratio. In other words, nominal wage stickiness can be stabilizing if monetary policy is used for stabilization and nominal interest rates are subject to a zero lower bound, a position long advocated by Keynesians.

## 4.2 Simulations with induced changes in policy

The simulation in figure 5 raises the investment parameter $\rho_1$ to 1.2, keeping all other parameters as in table 1. The initial values are $e_0 = 0.92, l_0 = 0.51$, with all other state variables at the stationary values associated with $E_1$. The increase in $\rho_1$ strengthens the destabilizing Harrodian forces, and the trajectories converge to a limit cycle. Asymptotically the interest rate becomes stuck at the ZLB, but the other other variables fluctuate; the employment rate, for instance moves between 0.83 and 0.95. Figure 5 shows bivariate patterns for $(e, \sigma), (e, \hat{p}), (e, r)$ and $(e, \omega)$.

Figure 5 about here

This outcome will almost certainly lead to changes in the policy rules. Central banks will react to a state in which their policy instrument ceases to be

[27] Similar qualitative results hold as long as an increase in debt has expansionary effects. This happens if $\rho_4$ is below the propensity to consume out of wealth. The two parameters are equal in the simulation depicted in figure 3a, which removes the feedback from the debt level to the other variables in the system.

[28] Exogenous limits on labor productivity productivity and the utilization rate of capital must curtail this process at some point. When that happens, the goods market can no longer clear through movements in output and productivity. Before it happens, however, there will almost certainly be changes in economic policy and private-sector behavior. The simulations have not imposed these upper limits.

[29] Interactions between fiscal and monetary policy are discussed by Bell-Kelton and Ballinger (2008), Ryoo and Skott (2017), Franke (2019), Mason and Jayadev (2018) and Franke (2019).

effective, and recurrent states of deep recession and significant overheating put pressure on fiscal policy makers to respond more aggressively. A likely result is increases in both the transfer parameter $t_2$ and the discretionary policy parameter $\gamma_3$. These changes are stabilizing. If $t_2$ is raised to 0.1 and $\gamma_3$ to 200, for instance, $e$ will fluctuate between 0.875 and 0.925, rather than between 0.83 and 0.95.[30] An alternative (or supplementary) change could be to let fiscal policy respond directly to inflation, especially when interest rates are at the zero lower bound and monetary policy has become ineffective; this innovation – making government spending depend inversely on the inflation rate – reinforces the stabilizing effect.

The reduction in the amplitude of the fluctuations has an important implication: the nominal interest rate moves off the zero lower bound for most of the cycle, and monetary policy regains traction. Thus, stability can be obtained by combining the fiscal changes with increases in the adjustment speed to $\lambda_r = 3$ and the sensitivity of the target interest rate to inflation to $\theta_1 = 4$. These changes in the monetary policy parameters would have been ineffective, by contrast, if the fiscal rules had not changed; the amplitude of the limit cycle would still be as in figure 5. Figure 6 illustrates the case with $t_2 = 0.1, \gamma_3 = 200$ and an unchanged monetary policy Figure 7 adds a more aggressive monetary policy ($\lambda_r = 3, \theta_1 = 4$); the convergence to $E_1$ is slow, and only the first 100 periods are shown. The initial values in figures 6 and 7 are the same as in figure 5.

Figures 6 and 7 about here

# 5 Conclusion

The analysis in this paper has pursued themes that go back to Karl Marx and Roy Harrod: locally unstable steady growth paths – a likely outcome in capitalist economies – form the basis for a theory of endogenous growth cycles; that is, for the integration of business cycles and economic growth. Unlike in DSGE models and other theories that take the steady growth path as intrinsically stable, there is no need for exogenous shocks to generate deviations from steady growth and for imposing particular structures of autocorrelation and time-varying volatility of these shocks to match the cyclical patterns in the data.[31]

The irregularity of observed business cycles is sometimes cited as evidence against endogenous cycles.[32] This argument is weak: if shocks are added to

[30] The COVID pandemic may be special, but the fiscal response in the US in 2020-2022 suggests a fiscal change of that order of magnitude.

[31] See Romer (2016) for a scathing critique of the cocktail of esoteric shocks and their role in DSGE models.

[32] Summarizing the prevailing views, Romer (2018, pp. 190-191) comments that

> Because output movements are not regular, the prevailing view is that the econ-

models of endogenous cycles, the cycles lose their regularity. The issue that separates the two approaches concerns the local stability of the steady growth path in the absence of shocks, a question that is orthogonal to the source and magnitude of exogenous shocks.[33]

The models in this paper assume that employment and the capital stock respond gradually to signals from the goods and labor markets. Labor productivity (the utilization rate of labor), however, is taken to be perfectly flexible, allowing output adjustments to clear the goods market in the short run.[34] Using empirically based behavioral functions and policy rules, the simulations of the extended model generated cyclical patterns that are consistent with US evidence.[35]

A stripped-down version of the model without a public sector also matched many of the features of US data. Like many nonlinear dynamic models, however, the properties of this version are very sensitive to changes in parameter values: plausible parameter values can reproduce many empirical patterns, but other equally plausible parameter generate very different outcomes, including cumulative divergence. This fragility of the results represents a serious weakness if the model is to be applied directly to real-world economies: there are no good reasons why private decision makers, acting to promote their own individual interests, would choose behaviors that produce the observed macroeconomic patterns.

The fragility of the qualitative results of the stripped-down model suggests that, except by a fluke, unregulated capitalism will be likely to descend into chaos and collapse. This implication may appear to be at odds with the evidence. In fact, however, there is no evidence to support or reject this implication: unregulated capitalism has never existed as a dominant mode of production. It has coexisted with non-capitalist sectors, mainly agricultural sectors in its early stages but with the public sector becoming increasingly important in later stages.[36]

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omy is perturbed by disturbances of various types and sizes at more or less random intervals, and that those disturbances then propagate through the economy. Where the major macroeconomic schools of thought differ is in their hypotheses concerning these shocks and propagation mechanisms.

[33] Econometric work based on linear specifications may also seem to favor local stability. But if, in fact, the data have been generated by a nonlinear system with local instability and bounded fluctuations, the conclusions from linear regressions will be biased, tending to support parameter values that imply local stability. The stability conclusions are easily reversed when the estimation allows for nonlinearities (Beaudry et al. (2017, 2020)).

[34] The economy may contain both flex-price and flex-output sectors. Skott (2023, chapter 10) analyzes a flex-price model with a public sector and, perhaps surprisingly, the properties and patterns predicted by the two models are quite similar. Thus, the models may also provide a good starting point for analyzing an aggregate economy that contains both types of sectors.

[35] The cyclical patterns in smaller and more open economies tend to be less regular, a finding that is not surprising: the movements of macroeconomic aggregates will not be governed by purely domestic interactions in economies that are strongly influenced by international trade and capital mobility.

[36] A recent literature on autonomous demand and supermultipliers has emphasized the instability of capitalist economies and the potential stabilization from non-capitalist sectors; see

The extended model includes empirically motivated modifications of investment behavior and, following Flaschel and his coauthors, of wage and price setting. More importantly, it adds a public sector with stylized fiscal and monetary policy rules. This addition greatly enhances the robustness of the qualitative outcomes: minor changes in parameter values no longer lead to global divergence, even if the policy rules are kept constant.

Policy rules are not constant, however. Unlike the production, investment and consumption decisions by individual firms and households, furthermore, they are chosen and implemented centrally. The political process behind economic policy is complex, and there can be no presumption of optimal rules, even if 'optimality' could be defined meaningfully in a world of uncertainty and conflicting interests. But it should be uncontroversial to suggest that deep depressions or explosive inflation lead to changes in economic policy. New policy tools may be devised – central banks reacted to the financial crisis and a binding zero lower bound by adding quantitative and qualitative easing to their toolbox – and traditional tools may be used more forcefully, as illustrated by the fiscal stimulus after the financial crisis and during the COVID pandemic as well as by the willingness of Japanese policy makers to allow large increases in public debt when a stagnating economy needed persistent stimulus.[37]

These observations weaken critiques of nonlinear dynamic models that point to potentially large differences in outcomes following a change in private behavior, keeping constant all policy rules. The prevailing policy rules are not independent of private sector behavior. Had private sector behavior been different in a way that significantly affected outcomes, policy rules would probably also have been different.

# Appendix A: Local instability of the baseline model

Short-run macroeconomic analysis assumes that the equilibrium levels of output and employment depend positively on the level of investment. The level of employment is predetermined in the baseline model, but a positive shock to investment initiates an adjustment process: output and employment expand, and standard short-run models assume that this happens so quickly that the capital stock can be taken as fixed. Thus, during the adjustment process we must have

$$d\hat{L} >> d\hat{K} \tag{36}$$

Now suppose that the economy is initially at a stationary point with $y = y^*, l = l^*$ and consider the implications of a marginal increase in $l$. With a given capital stock $K$, the increase in the employment capital ratio $l$ reflects a rise in $L$ :

$$dL = Kdl$$

e.g. Freitas and Serrano (2015, Allain (2015, 2021), Lavoie (2016). The distinctive ways in which this insight has been formalized in this literature is questionable, however; Skott (2019), Skott et al. (2022).

[37] Nakatani and Skott (2006) and Skott (2023, chapter 11) discuss the Japanese case.

The increase in the labor capital ratio also affects both investment and the short-run equilibrium level of employment (the latter associated with $y = y^*$ and $Y = y^* L$):

$$\begin{aligned} dI &= f' K dl \\ dY^{eq} &= y^* dL^{eq} = m dI = m f' K dl \end{aligned}$$

where $m, Y^{eq}$ and $L^{eq}$ are the investment multiplier and the short-run equilibrium levels of output and employment. If it takes $T$ periods for the multiplier to work itself out, with $T$ so short that the capital stock can be taken as constant, we have

$$d\hat{L} = \frac{d\frac{L}{K}}{\frac{L}{K}} \approx \frac{1}{l^*}\frac{1}{T}(\frac{L^{eq} - L}{K}) = \frac{1}{T}\frac{m\frac{f'}{y^*} - 1}{l^*} dl$$

Now return to the model in equations (10)-(11) and observe that the trace of the Jacobian can be written[38]

$$\begin{aligned} Tr &= l^*(\frac{d\hat{L}}{dl} - \frac{d\hat{K}}{dl}) + e^* \frac{\partial \hat{L}}{\partial e} \\ &= y^* l^* (\frac{1}{T}\frac{m\frac{f'}{y^*} - 1}{y^* l} - \frac{f'}{y^*}) + e^* h_3 \qquad (37) \end{aligned}$$

Hence, with a multiplier well above one and a strong sensitivity of accumulation rates to changes in employment capital ratio in the neighborhood of the stationary point, rapid convergence to short-run equilibrium (a small value of $T$) is destabilizing: the first term on the right hand side of equation (37) will be positive, and local stability requires a strong negative effect from employment to the growth rate of output.[39]

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[38] The notation $d\hat{L}/dl$ is used here to indicate the ‘partial total derivative’: $d\hat{L}/dl = h_1 y' + h_2$.

[39] The value of $y^*$ at the stationary solution depends on the choice of units; measuring labor input in terms of days rather than hours, for instance, will increase the value of $y^*$. But the term $y^* l^* (\frac{1}{T}\frac{m\frac{f'}{y^*} - 1}{y^* l} - \frac{f'}{y^*})$ is independent of the choice of units: investment reacts to sustained changes in the utilization rate of capital, and a change from hours to days will reduce $l^*$ and increase $f'$, leaving both $y^* l^*$ and $f'/y^*$ unchanged.

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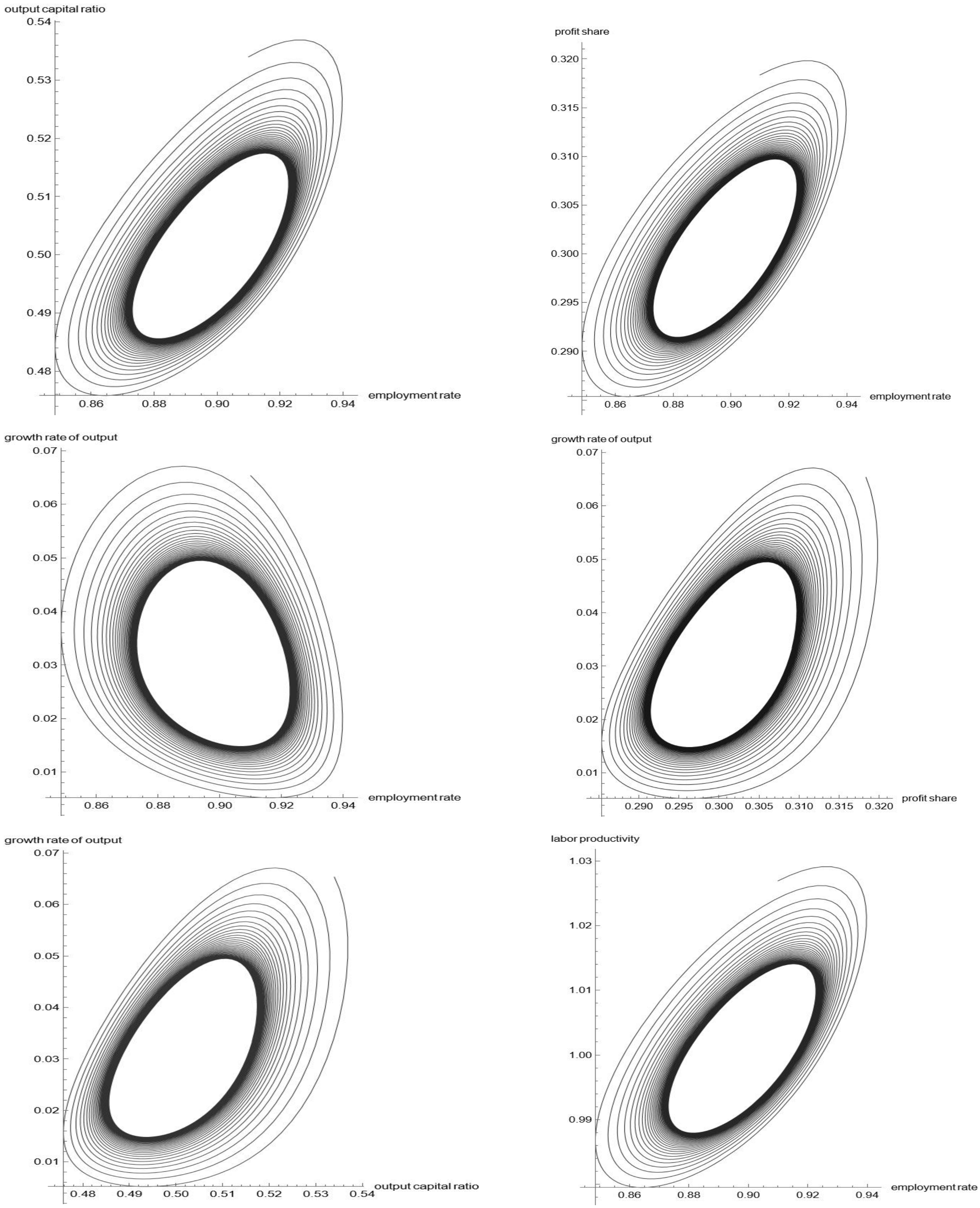


**Figure 1a: Limit cycle in simulation of baseline model (investment parameter = 1)**

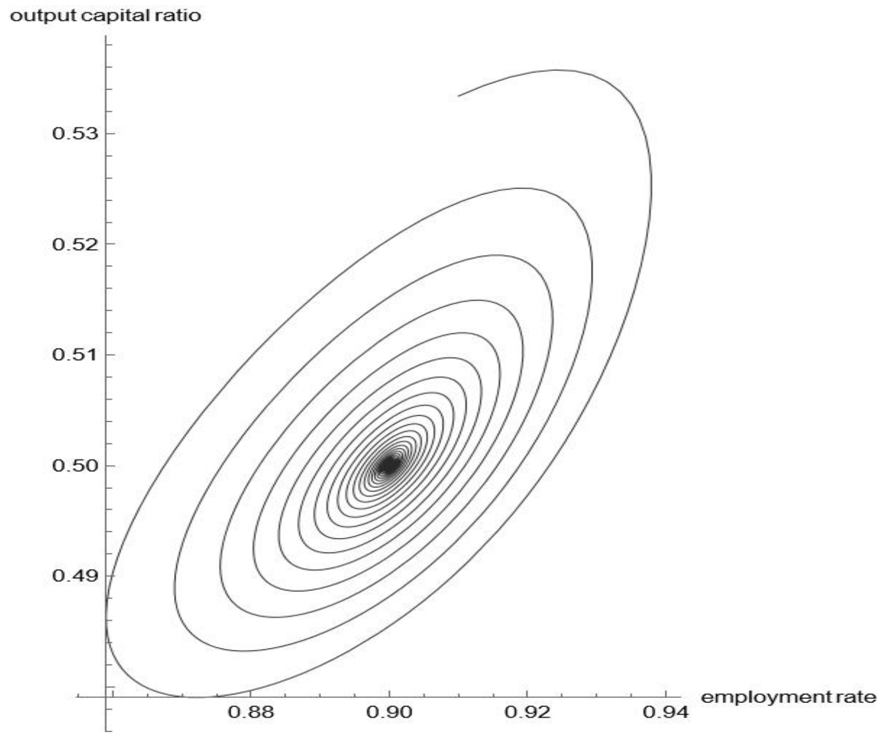


**Figure 1b: Simulation of baseline model with local stability (investment parameter = 0.97)**

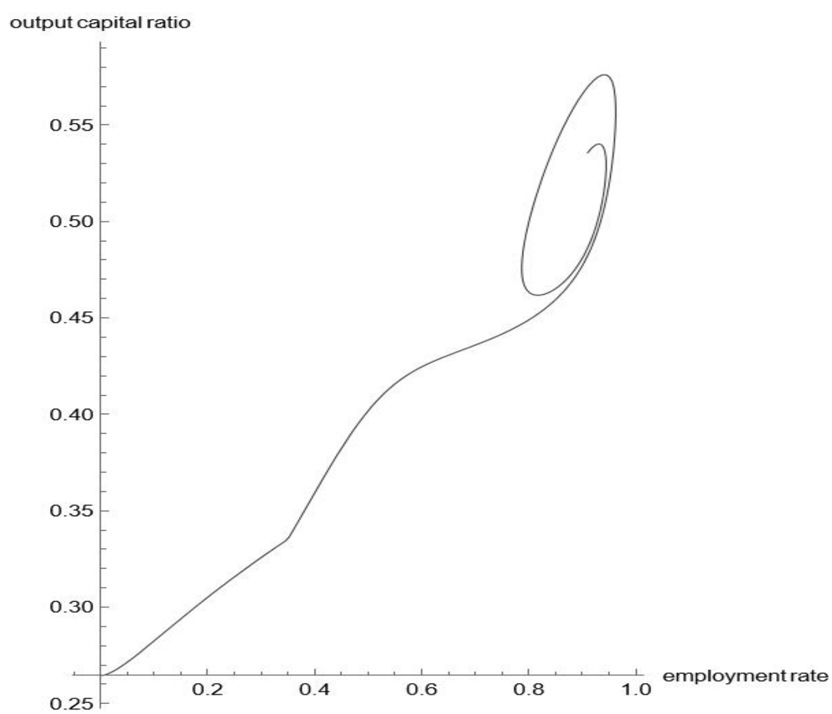


**Figure 1c: Simulation of baseline model with divergence (investment parameter = 1.08)**

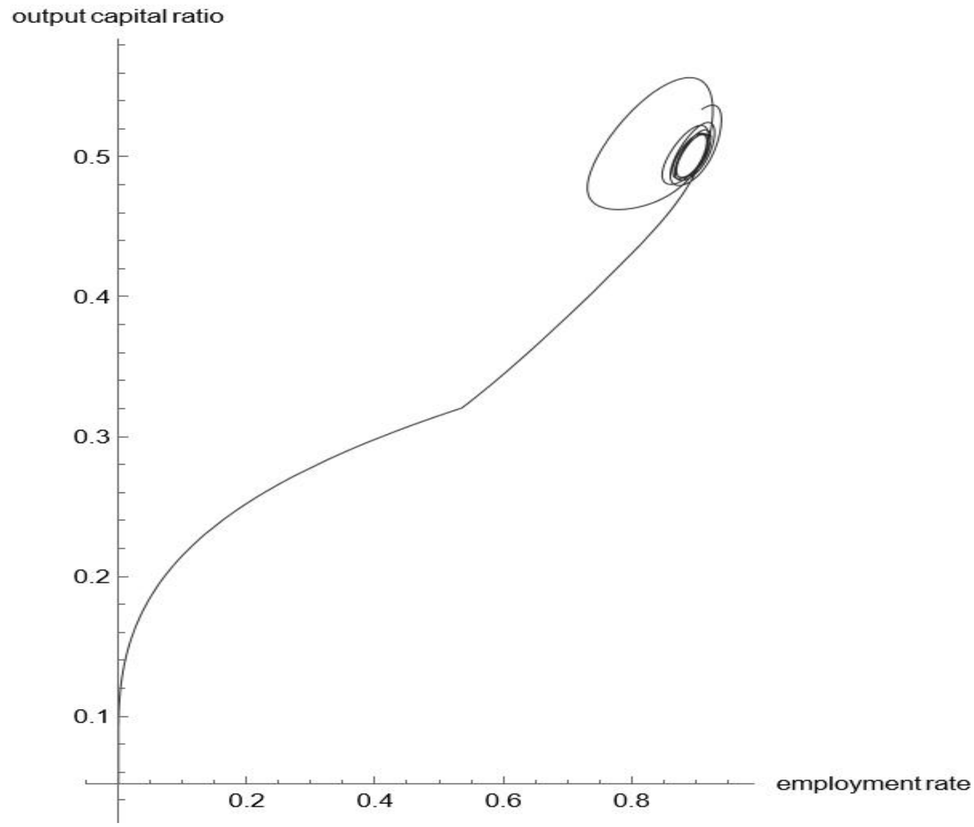


**Figure 2b: Simulation of baseline model with endogenous wage growth ($\alpha_1 = 0.01, \alpha_2 = 0$)**

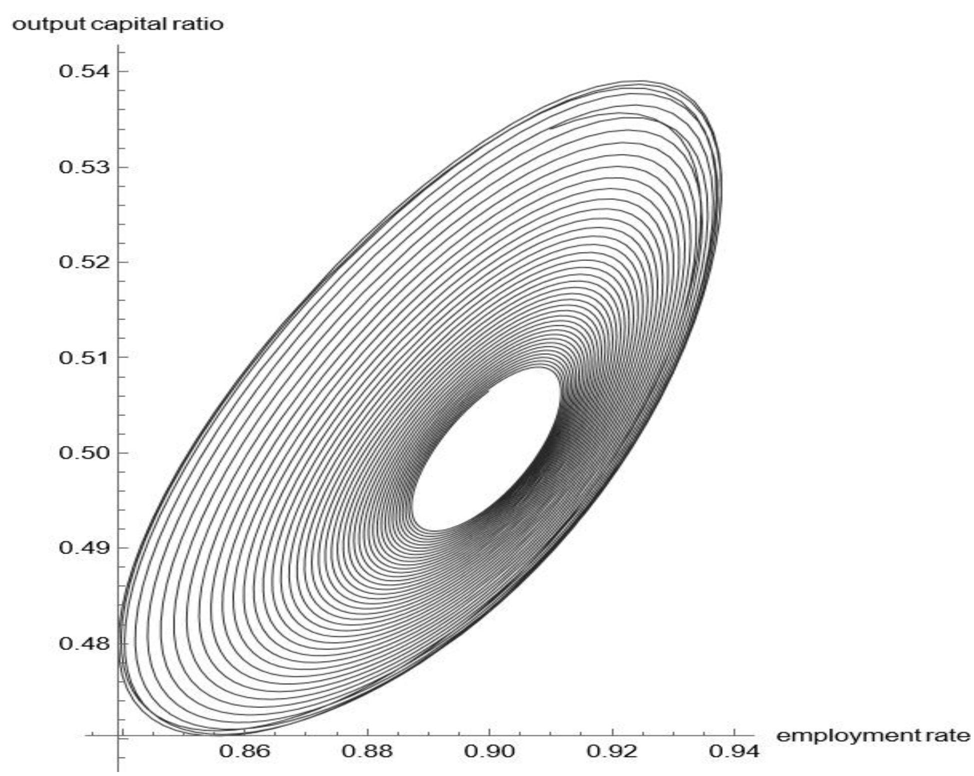


**Figure 2b: Simulation of baseline model with endogenous wage growth ($\alpha_1 = 0, \alpha_2 = 0.1$)**

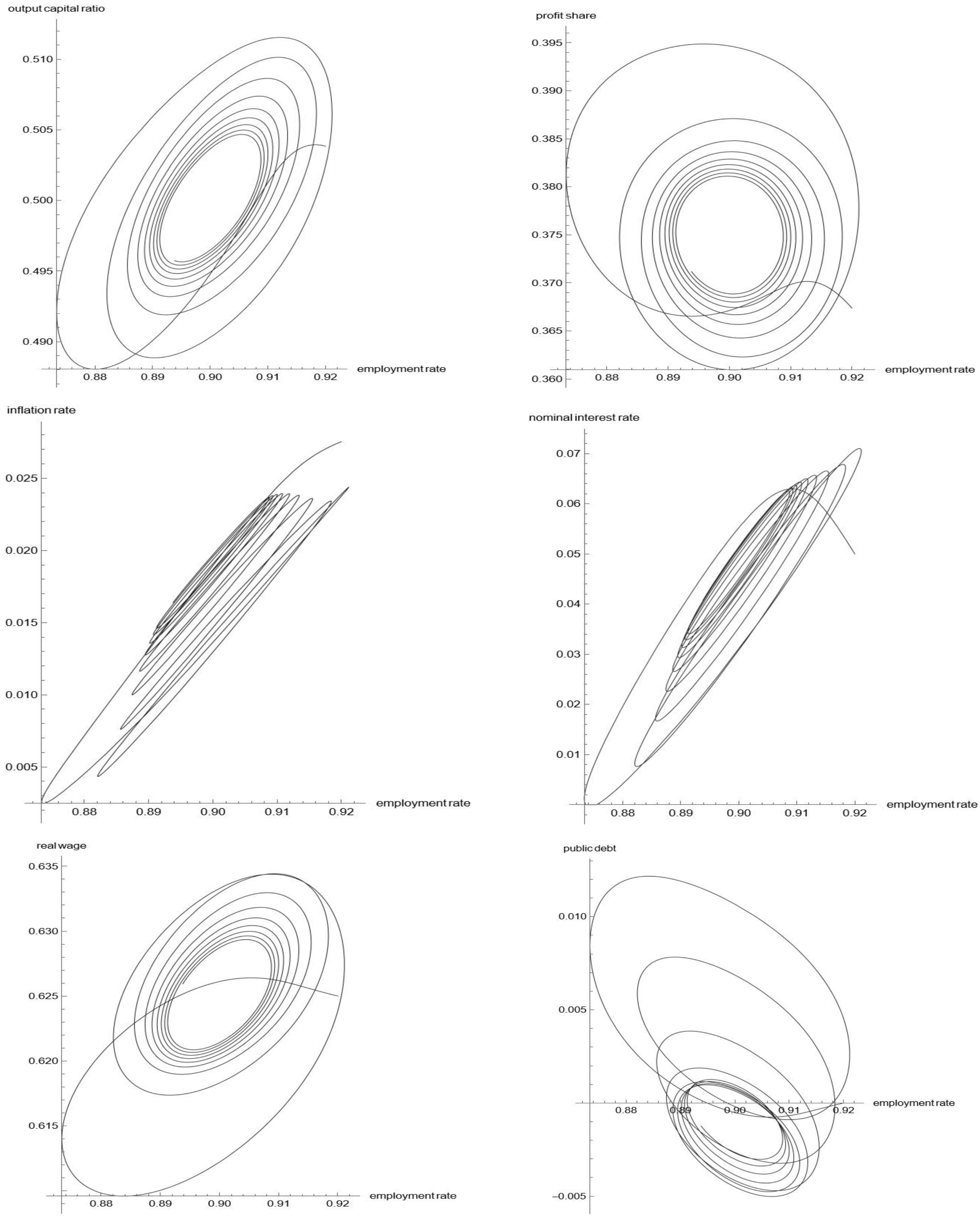


**Figure 3: Extended model with slow convergence ($\rho_1 = 0.89$)**

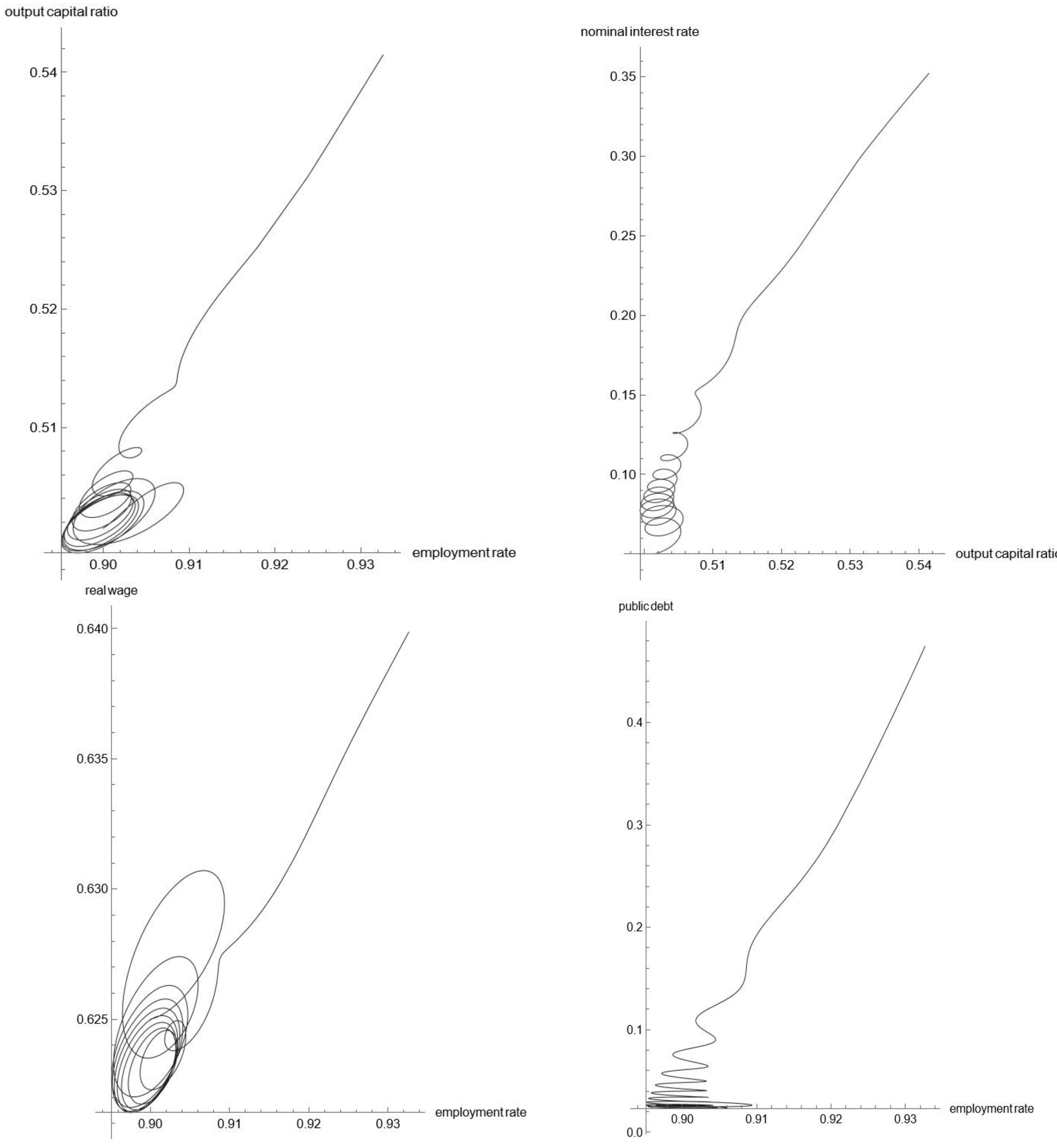


**Figure 4: Extended model without feedback from government debt ($\gamma_4 = 0$)**

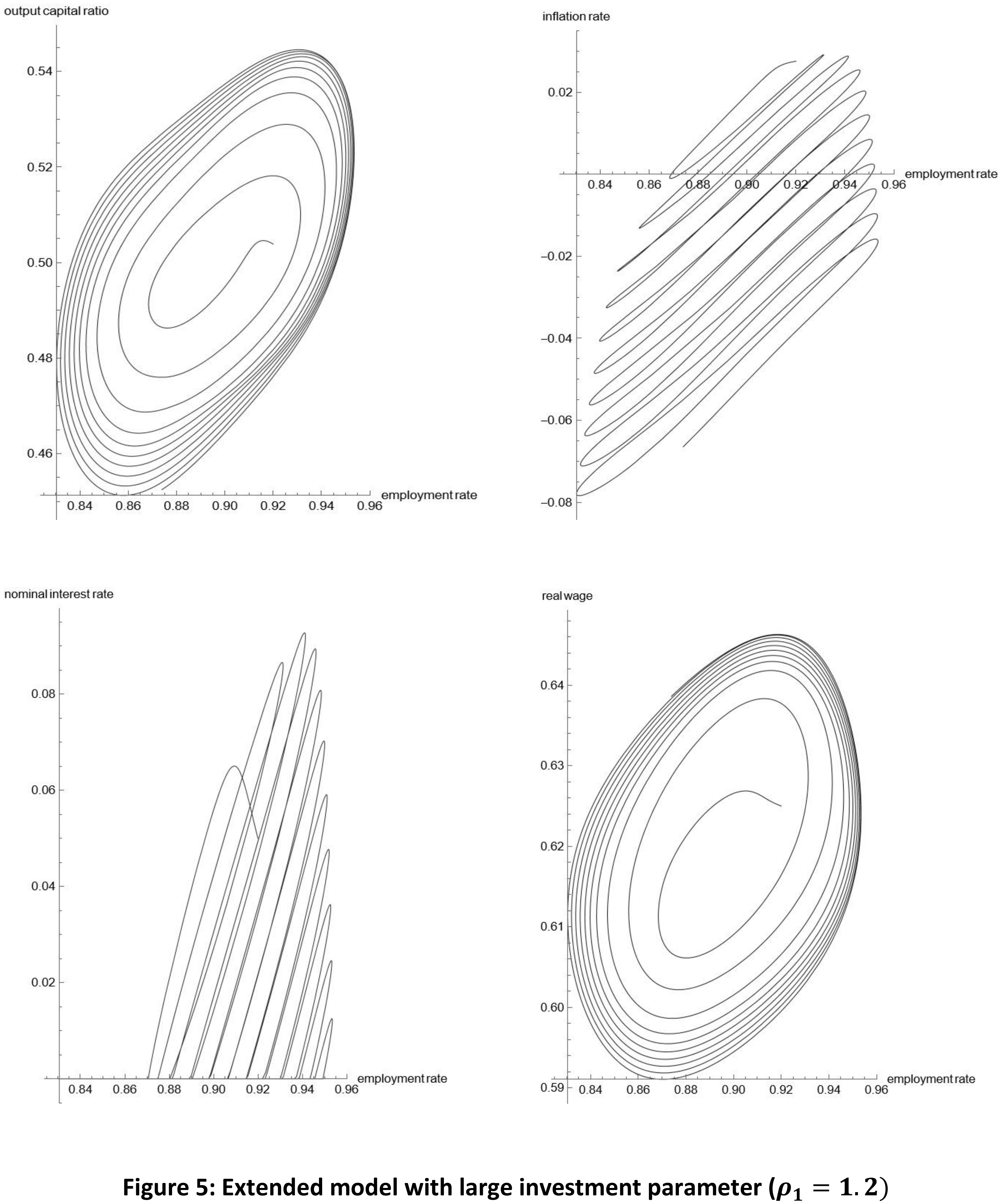


**Figure 5: Extended model with large investment parameter ($\rho_1 = 1.2$)**

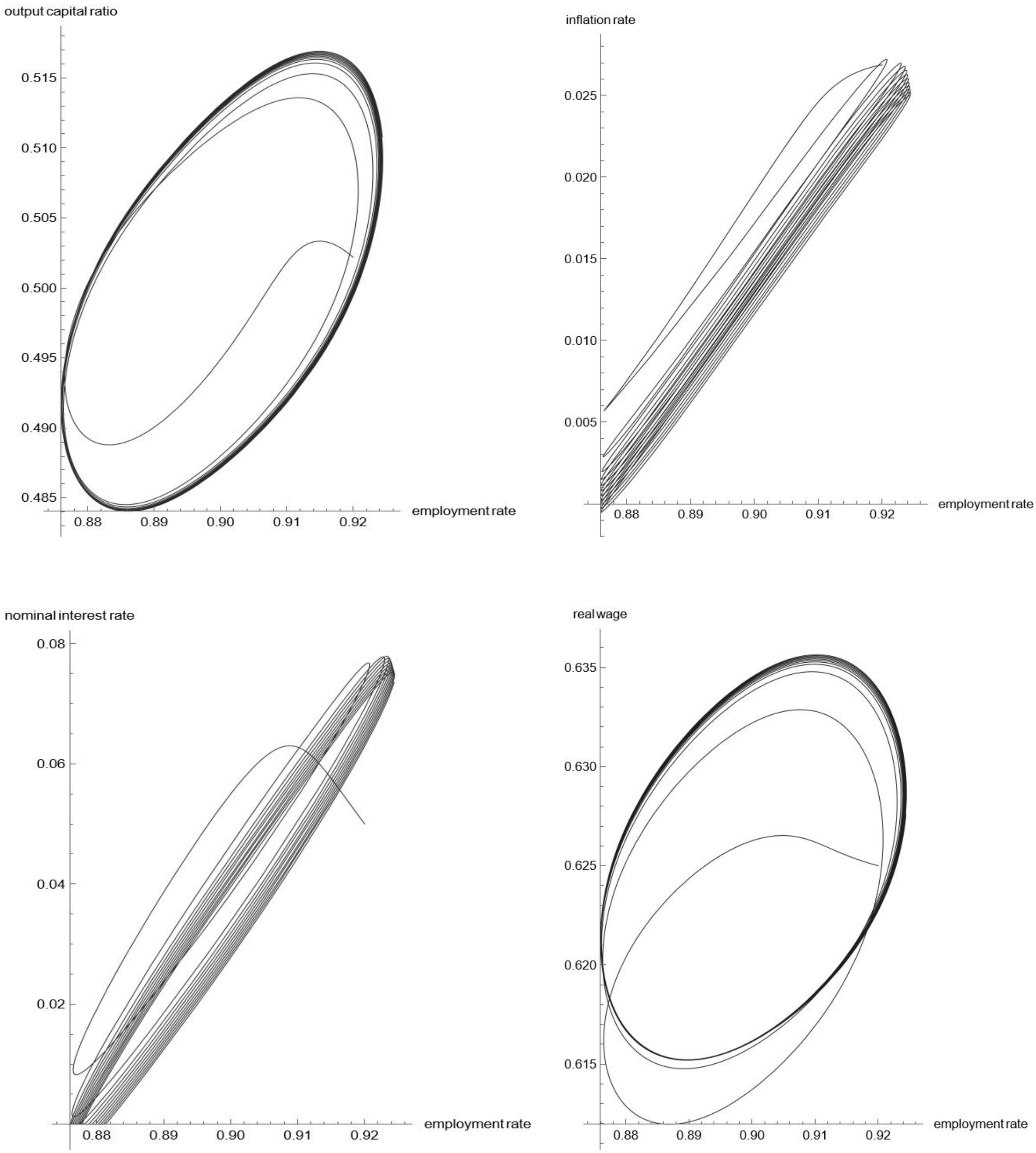


**Figure 6: Extended model with large investment parameter and induced change of fiscal policy ($\rho_1 = 1.2, t_2 = 0.1, \gamma_3 = 200$)**

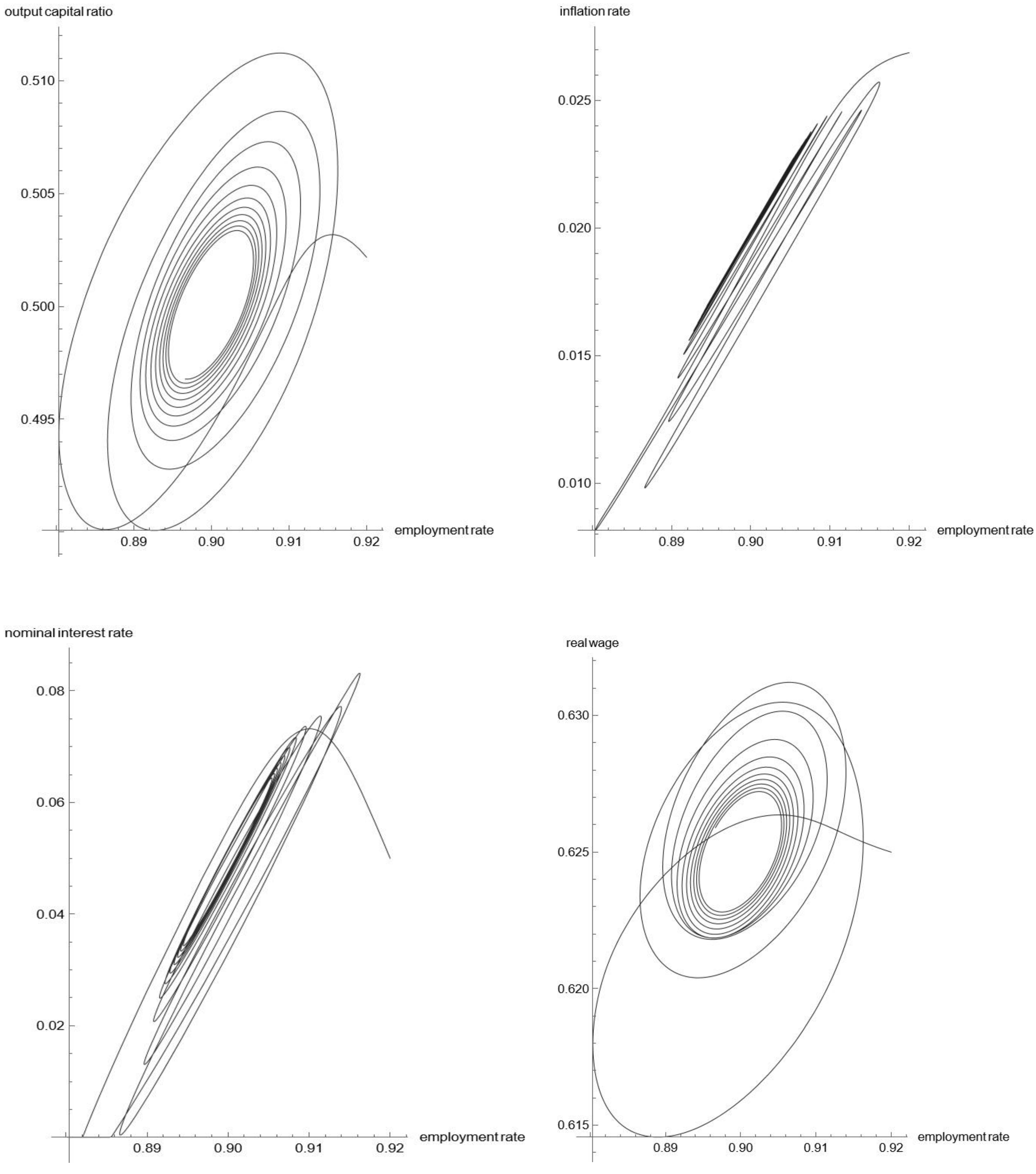


**Figure 7: Extended model with large investment parameter and induced changes of fiscal and monetary policy ($\rho_1 = 1.2, t_2 = 0.1, \gamma_3 = 200, \lambda_r = 3, \theta_1 = 4$)**