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By Ulrich van Suntum

and Tom Neugebauer

Böhm-Bawerk meets Keynes – what does determine the interest rate, and can the latter become negative?

Zusammenfassung

Auch während des 100. Begehen von Böhm-Bawerks Todesjahr und nahezu 70 Jahre nach dem Tod von Keynes sind die langfristigen Bestimmungsgründe des Zinses noch immer umstritten. Den realwirtschaftlichen Theorien der österreichischen Schule stehen die vorwiegend monetären keynesianischen Erklärungsansätze scheinbar unversöhnlich gegenüber. Das führt dazu, dass die aktuellen Niedrig- und Negativzinsen von prominenten Ökonomen nach wie vor unterschiedlich betrachtet und erklärt werden. Viele sehen sie als direkte Folge der expansiven Geldpolitik der Notenbanken, während dagegen andere auf einen Kapitalangebotsüberschuss in den alternden Industriegesellschaften verweisen. Der vorliegende Beitrag kombiniert zentrale Aspekte dieser verschiedenen Zinstheorien im Rahmen eines Strom-Bestandgrößen-konsistenten Makromodells und untersucht deren Wirkung auf die Bestimmung des Zinses. Es wird gezeigt, dass sich die Erklärungsansätze in der Tradition von Böhm-Bawerk und Keynes keineswegs ausschließen, sondern einander gut ergänzen.

Summary

100 years after the death of Böhm-Bawerk and nearly 70 years after the death of Keynes, there is still a fundamental disagreement about the factors that determine the interest rate in the long run. While Economists in the Austrian tradition see it as solely driven by real phenomena, Keynesian authors mainly stress monetary factors. Likewise, the current phase of low interest rates is explained in many different ways by prominent economists. While many economists blame the expansive monetary policy to be the reason for it, other economists point to the excess capital supply in ageing industrial states. By setting up a coherent stock-flow consistent macro model of a closed economy, the present paper combines central aspects of these explanations to analyze their respective influence on the determination of the interest rate. It is argued that theories in the tradition of Böhm-Bawerk and Keynes respectively do not at all preclude each other, but, on the contrary, complement each other and help to understand the current low interest rate puzzle.

JEL-Klassifikation: E10, E40, E50
JEL-Schlüsselwörter: public debt, stock-flow consistent model, monetary policy

I. Introduction

100 years after Böhm-Bawerk's death and 70 years after the death of Keynes, there is still a fundamental disagreement about the factors that determine the interest rate in the long run. While Economists in the Austrian tradition see it as solely driven by real phenomena (Block (1999), Huerta de Soto (2006)), Keynesian authors mainly stress the monetary factors. Likewise, the current phase of low interest rates is explained by prominent economists, differently. While many economists blame the central bank's expansive monetary policy for the low interest rate (Bracke and Fidora (2008)), others economists are in favor of the saving glut hypothesis that points to the excess capital supply in ageing industrial states (Bernanke et al. (2011) and von Weizsäcker (2013)).

We argue that the low interest rate is best explained by a combination of Böhm-Bawerk's and Keynes' theory of the interest rate. To show that, we set up a small scale macroeconomic model similar to that of van Suntum (2013). The model is both stock-flow consistent and fully micro-founded. Stock-flow consistency means that there is no flow, like investment, without the corresponding stock change, real capital, in the model and vice versa. Pioneering work in this respect has been done by Tobin (1969), Taylor (2004), Godley and Lavoie (2007 I; II) and e Silva and Dos Santos (2011). First, this methodology supports our view that a change in the quantity of money via changes in the distribution of current income disturbs the optimal size and composition of household's wealth, and thereby influences the interest rate. And second, unlike in DSGE-models, stock-flow consistent models allow for more general results in the long run, since all important flows which may become stocks and vice versa are fully considered. In other words, stock-flow consistent models are presentable in terms of the System of National Account.[1]

In order to focus on the main mechanisms behind the determination of the interest rate, we have chosen to keep our model as simple as possible. It only consists of private households, firms, a central bank and a government.[2] Private households maximize an intertemporal utility function, in which not only consumption and savings, but also wealth in form of both real capital and liquidity are included. Private firms produce a single commodity (corn), which can be used for consumption and investment purposes alike. By this simplification we circumvent the problem of an unequivocal definition of capital in a world with heterogeneous goods, which had puzzled Böhm-Bawerk and many other authors working in this field (Cohen and Harcourt (2003)). Capital goods depreciate completely after one period, and thus vanish from period to period. Hence, problems with properly defining the roundaboutness of production are also absent in our model. Unlike van Suntum (2013), the existence of both a private bank sector and multiple capital market interest rates are neglected. Nonetheless, due to the existence of a central bank and a government, our model allows for an analysis of monetary impacts on the determination of the interest rate, as well as the impacts of changing public debt. Calculating steady states, we restrict our analysis to the examination of long-term steady results.

The remainder of this paper is organized as follows: In Section II, the model is introduced and Böhm-Bawerk's three causes for the existence of an interest rate are worked out and linked to it as well as Keynes' monetary concept of liquidity preference. Thereby, a simple micro-founded macro-model is created, in which both real and monetary determinants of the interest rate are brought together. In Section III, we derive the steady state equilibrium conditions and discuss the most important results concerning the determinants of the interest rate. In particular, the impact of monetary policy on the steady state in terms of the interest rate, total output, and the price level is investigated. Additionally, we compare our model with Samuelson's barter economy approach from 1958 and investigate under which conditions the interest rate can become negative. Section IV concludes and points to some limitations of our model.

[1] See Taylor (2008) and Papadimitriou and Zezza (2011).
[2] An important feature of our model is that it can be solved analytically which makes it perfect for teaching purposes at the undergraduate level in economics.

II. The model

1. Flow optimization by private households: Böhm-Bawerk's first two causes

In order to derive Böhm-Bawerks first two causes for the existence of an interest rate[3], we make use of a simple OLG-model, which is similar to Diamond's model from 1965.[4] Whereas the young work, and thereby receive income to accumulate wealth and to save for their later retirement period, the elderly only dis-save what they earned in their young life period. With respect to flows, the representative household maximizes the following logarithmic utility function, which is standard in modern textbooks[5]:

$$(1)\ln U_F = \frac{1}{2-\theta}\ln C_1 + \frac{1-\theta}{2-\theta}\ln C_2 \qquad \text{with } 0 \le \theta \le 1$$

C_i denotes the households consumption quantities in period $i \in 1;2$ of its life. The parameter θ indicates the household's rate of time preference. Note, that with $\theta = 0$, there could be still an incentive to antedate consumption, due to the diminishing marginal utility in each period. In particular, as Böhm-Bawerk rightly stated, consumption tends to be less valuable in the future than today, because the households expect their income to increase over time. Hence, starting with $C_1 = C_2$, households would try to shift some part of their consumption from period 2 to period 1, which in turn would immediately generate a positive rate of interest. This relative overvaluation of present income is nothing different than Böhm-Bawerks first cause.

Böhm-Bawerk's second cause points to a psychological law which states that future needs are systematically underestimated by the households (individuals). This is where the modern notion of time preference comes into play. Even if marginal utility were constant, a positive θ would create a tendency to antedate consumption. In the extreme case, in which future consumption is completely denied in favor of today's consumption, and, hence, the time preference rate would be one, $\theta = 1$, consumption would only occur in period 1. By considering the preferences of the households, Böhm-Bawerk introduced subjective reasoning as key aspects to the understanding of economic phenomena, like the interest rate. Subjective time preference became experimentally well documented and consequently implemented in modern economic theory (Fishburn and Rubinstein (1982) and Hoch and Loewenstein (1991)).

Hence, in our textbook-utility function two of Böhm-Bawerks main arguments are already incorporated. By de-logarithmizing, (1) can be transformed into the more convenient form:

$$(2) U_F = C_1^{1/(2-\theta)} C_2^{(1-\theta)/(2-\theta)}$$

We assume that households earn original income only when they are young. When they are old in their second life period, they just consume their wealth, which consists of savings as well as of earnings from interest payments. The respective budget constraints are given by:

$$(3) Y_{H1} = C_1 + S_1$$
$$(4) S_1(1+i_v) = C_2$$

S_1 denotes savings in the first period of an household's life and i_v is the average interest rate paid on savings.[6] Applying the Lagrange-method, the following set of optimal flows is obtained:

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[3] For a short overview see e.g. Ekelund and Hebert (2007), pp 313.
[4] Compare Diamond (1965).
[5] See for example Romer (2006), p. 51.
[6] Since only a part of the household's total wealth is held as liquidity, i_v is generally not equal to the (capital market) interest rate. Please, see below.

$$(5) C_1^* = \frac{Y_{H1}}{2-\theta}$$
$$(6) C_2^* = \frac{(1-\theta)(1+i_v)Y_{H1}}{2-\theta}$$
$$(7) S_1^* = \frac{(1-\theta)Y_{H1}}{2-\theta}$$

Note that according to (7), savings are independent from the interest rate and representable as a fraction of the income which only depends on the rate of time preference. Leaving optimization of private wealth to a later section, we leave the household sector for now.

2. *The productive sector: Böhm-Bawerk's third cause*

Unlike other Austrian economists[7], Böhm-Bawerk also adjudged that the production side of the economy influences the determination of the interest rate. He derived his third cause from the roundaboutness of production, thereby creating a temporal capital theory that has many similarities to that of the German economist Heinrich von Thünen.[8]

In our model, we acknowledge this issue by incorporating a most simple production function where capital is the only factor of production:

$$(8) Y = K_F^{\beta}$$

Y is gross domestic production, K_F is real capital (seed), and β is the relevant elasticity, which is assumed to be less than unity. Hence, the production function possesses positive, but diminishing marginal returns on capital, as it is generally assumed in neoclassical production theory as well. Since, we assume that real capital (seed) completely vanishes in one period, the production function has to cover both for capital regeneration and interest, simultaneously. The income is distributed between the households and the capital market as follows:

$$(9)(1+i)K_F = \beta Y$$
$$(10)\Pi_F = (1-\beta)Y = Y_{H1}$$

i is the capital market interest rate and Π_F are the firm's profits which accrue to the households when they are young. As an illustration, one can conceive that the households both operate the farms and supply them with capital in the first period of their life, while just reaping the fruits of their savings after retirement.

It is worth noting that with $0 < \beta < 1$ we have $Y > K_F$ if $K_F < 1$ and vice versa. That means that the net rate of return may turn negative if the gross output *Y* is lower than the capital input. We will come back to this issue when we will discuss the conditions for a negative interest rate.

3. Private wealth optimization: Introducing money and bonds

Let us return to the households and examine how they optimize their stocks. In our model the household's wealth consists of both bonds and liquidity. Bonds are normally issued either by firms, by the government, or by the central bank. By purchasing bonds, households provide capital K_H to the capital market and receive the respective interest rate i. It is assumed that bonds have a maturity of just one period and, hence, have to be renewed every single period.

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[7] See for example Fetter (1914).
[8] See for example Samuelson (1983) and Etula (2008).

Liquidity in the form of paper money is held by private households out of two motives that have already been noted by Keynes. On the one hand, money is needed to carry out daily market transactions. This is namely the transaction motive of holding money. And on the other hand, money can also be held as a luxury good, which is used for speculative purposes or just as a liquid form of wealth. This corresponds to the speculative motive of holding money. Viewing the transaction motive as a pure need which does not autonomously add to the individual´s wellbeing, only idle money,(L_S / p), is considered to be a part of the household's utility function in our model. Nonetheless, the transaction money,(L_T / p), as part of the household's wealth, will become later important for the determination of the price level. Another important aspect for defining it that way is that there would be double-counting, since the consumer goods have already been recognized in the flow-part of our utility function (1).

Formally, we assume the following stock-part of the utility function:

$$(11) U_S = (K_H)^i (L_S / p)^l$$

K_H is the household's capital supply and L_S / p is that part of liquidity which is held in excess of pure transaction needs (Keynesian idle balance). Both are defined in real terms and weighted by their respective rate of return which is the capital market interest rate i in case of K_H and the non-pecuniary advantage of liquidity l in case of L_S .

Total utility is assumed to be simply the sum of (1) and (11):

$$(12) U = U_F + U_S$$

Due to this specification, there is no problem with maximizing the two parts separately. The volume of $V = K_H + (L_S / p) = S$ is known from (7). So, by employing (10) in addition, we can easily maximize (11) in order to get the optimal structure of individual wealth:

$$(13) K_H^* = \frac{i}{i+l}\left(\frac{1-\theta}{2-\theta}\right)(1-\beta)Y$$

$$(14) (L_S / p)^* = \frac{l}{i+l}\left(\frac{1-\theta}{2-\theta}\right)(1-\beta)Y$$

Obviously, according to (13) and (14), household's save more wealth in the form of bonds, the higher the interest rate is, and tend to hold the more liquidity, the higher the respective non-pecuniary advantages are. These findings are in accordance with both intuition and Keynesian liquidity preference theory.

The average interest on savings which we have referred to in (4) above can be calculated as follows:

$$(15) i_v = \frac{iK_H}{K_H + L_S / p} = \frac{iK_H}{V}$$

With this supplement of household's utility maximizing in terms of stock, two goals have been achieved: First, like in Böhm-Bawerk's theory, but in contrast to many other theories, household's stocks in addition to household's flows play a role in the explanation of the interest rate, as it was rightly demanded by Tobin (1969). Second, unlike Böhm-Bawerk, we have created a link between real and monetary determinants of the interest rate as well as between Austrian and Keynesian theory.

4. The public sector

Being the monetary authority, liquidity is brought into circulation by the central bank. We limit our analysis exclusively on open market policy. That is, the central bank purchases bonds K_M (defined in terms of commodity units) at the capital market in order to create money. In addition, we assume that there is some initial amount of paper money, $\overline{M}$, which is defined in nominal terms. It greatly facilitates calculations when K_M is defined as a fraction of total income, namely m . Consequentially, total money supply is given as:

$$(16) M = \overline{M} + mpY$$

Like any other provider of capital, the central bank earns interest payments iK_M. Since the central bank does not consume anything, it distributes all of these payments to the government that consume it, entirely. For the sake of simplicity, these payments are assumed to be the only source of public receipts.[9] However, the government can also take credit at the capital market by issuing public bonds K_G which are defined in commodity units, too. Again, we define the volume of these bonds as a fraction of total income, namely g . Summarizing, the government receipts are either consumed or required for paying interest payments on public debt, thus, the steady-state government's budget is given by:

$$(17) i(K_M - K_G) = iY(m - g) = C_G$$

Note that public debt reduces public consumption in the steady state unless the interest rate is negative[10]. Only in the transition period, which is not in the scope of the present paper, the government can extend its primary expenses by taking additional debt as it is well known from conventional growth theory.

III. Equilibrium conditions and steady state results

1. *Equilibrium interest rate and price level*

To make our model work, we first have to derive the steady state equilibrium conditions. With respect to the capital market, the following condition has to hold:

$$(18) K_H + K_M = K_F + K_G$$

Taken into words, capital supply that is at the left hand side of (18) has to equal total capital demand. By implementing (9), (13) and (17) into (18), the condition can be written as:

$$(19) \frac{i}{i+l}\left(\frac{1-\theta}{2-\theta}\right)(1-\beta)Y + mY = \left(\frac{\beta}{1+i}\right)Y + gY$$

After cancelling Y in (19) and some more manipulation, the following quadratic equation for the interest rate is derived:

[9] In van Suntum's model (van Suntum, 2013), the government's budget is the total of the central bank profits and tax payments by the households, which accrue due to a proportional tax on their respective income. Since a proportional income tax does not change our findings qualitatively, it is omitted.

[10] More precisely, this is the case unless the economy's growth rate (which is zero in our model) exceeds the interest rate.

$$(20)\, i = -\frac{A_1}{2} \pm \sqrt{\left(\frac{A_1}{2}\right)^2 - A_2}$$

with

$$A_1 \equiv \frac{\frac{(1-\theta)}{(2-\theta)}(1-\beta) - \beta + (m-g)(1+l)}{\frac{(1-\theta)}{(2-\theta)}(1-\beta) + m - g}$$

$$A_2 \equiv \frac{(m-g-\beta)l}{\frac{(1-\theta)}{(2-\theta)}(1-\beta) + m - g}$$

Obviously, all variables in (20) are exogenous, so both the interest rate and all other endogenous variables in real terms can easily be calculated.

Concerning nominal terms, we further need to know the commodity price level p , which can be calculated from the monetary equilibrium condition. The latter is given by:

$$(21)\; M/\,p \;=\bar{M}/\,p + mY = (L_S\,/\,p) \;+\; (L_T\,/\,p)$$

This condition states that the real money supply, $\bar{M}/\,p + mY$, has to be equal to the total real liquidity demand of the households, consisting of idle liquidity $(L_S\,/\,p)$ and liquidity held for transaction purposes $(L_T\,/\,p)$. From (21) the price level can easily be derived as:

$$(22)\, p = \frac{\bar{M}}{(L_S\,/\,p) + (L_T\,/\,p) - mY}$$

Some numerical examples of the whole set of steady state equilibrium values for all variables are found in the appendix. In the following, we do not provide any formal proofs, but restrict ourselves to highlight some very general results that have been derived by both numerous simulations and economic intuition. Most of them are also easily verified by a quick inspection of the relevant equations.

2. *What factors actually determine the interest rate and how?*

Besides all three of Böhm-Bawerk's causes for the existence of an interest rate, we find that both fiscal and monetary policy have an influence on the determination of it as well. Formally, these findings can be derived from (20), where neither g nor m cancel out. On the other hand, $\bar{M}$ does not appear in (20), while it is proportionate to the price level according to (21). Therefore, (21) seems to support the widespread believe that a pure increase in the quantity of money does only lead to inflation, but does not affect the interest rate, at least not in the steady state.

Indeed, one has to be quite careful here. A simple multiplication of money by helicopter would actually only increase the price level. The same is true for the – perhaps more realistic case – that the central bank purchases any commodities for freshly printed notes. So far, the Austrian view is supported by our model.

However, things are different, when money enters the economy as debt money, as it is the normal case in real world economies. That important issue has already been detected by Metzler (1951), who noted that the central bank does not only print notes, but furthermore acts as a supplier of capital at the market. Due to the missing coverage by real goods, no real savings stand behind the central bank's capital offer, so that the additional supply of money can thoroughly be seen as some sort of cheating. Nevertheless, this money supply decreases the interest rate, even in the long run according to (20).[11] This would not be the case, if only households remembered that the central bank is lastly their own. That means that with a given amount of public goods, its profits tend to reduce their tax burden by the same amount. In this case of Ricardian equivalence they would namely reduce their own savings, which would ideally lead to $\Delta K_M = -\Delta K_H$, meaning that no impact of monetary policy on the interest rate would occur. However, it is not very likely that Ricardian equivalence in this extreme form really exists. Therefore, the central bank can reduce the interest rate, principally, by whatever extent, and even in the steady state. A decrease of the interest rate would ceteris paribus increase both the capital stock and total output in the economy, which follows from (9).

	Low transaction liquidity – income ratio			High transaction liquidity – income ratio		
	Interest rate	Output	Price level	Interest rate	Output	Price level
dm > 0	-	+	+	-	+	-

Table 1: Effects of Monetary Policy

However, these benefits would generally go along with a change in the commodity-price level, according to our simulations. Depending on the value of exogenous variables, interestingly, either temporary inflation or temporary deflation is possible. Normally, one would solely expect a rising price level, due to the increase in money supply. But, there is also a real output increase and a decrease of the interest rate, caused by the extra demand for idle money. If the extra demand for idle money exceeds the interest rate, deflation rather than inflation will occur (see Table 1).[12]

Although this would be only a one-off effect, it cannot be neglected from a welfare point of view. Future generations would benefit from a permanently higher income and wealth. However, in case of inflation the generation in the transition period would suffer from a devaluation of their savings, while in the deflationary scenario debtors who live in the transition period would correspondingly suffer from an appreciation of their debts. Hence, no easy conclusions about a Paretian improvement in response to expansionary monetary policy can be made.

Concerning public debt taking, there is definitely an increase of the long term interest rate. Formally, this can be derived from (20). The reason for this is that we have assumed that all government receipts are consumed. So, an increase of public debt simply causes a crowding out at the capital market without any compensation in the form of public investment. Correspondingly, the higher public debt is in relation to total income, the lower is both total income and private capital stock (see Table 2).

	Low transaction liquidity – income ratio			High transaction liquidity – income ratio		
	Interest rate	Output	Price level	Interest rate	Output	Price level
dg > 0	+	-	+	+	-	+

Table 2: Effects of Fiscal Policy

It is also obvious what happens in the model when additional public debt is financed by additional debt money, for example in the case of monetizing public debt. Formally, the equally increasing g and m in (20) cancel out. In that case, no variable, but the price level which according to (22) definitely increases, changes in the steady state. The interest rate, total income and its distribution would all be the same as before in the long run. This finding also applies to the relative size of the public sector. Even though, the government now faces higher interest expenses, these expenses are exactly outweighed by the correspondingly higher central bank profits. Consequently, the government can neither spent less nor more for public consumption than in the initial steady state (see Table 3).

	Low transaction liquidity – income ratio			High transaction liquidity – income ratio		
	Interest rate	Output	Price level	Interest rate	Output	Price level
dm = dg > 0	0	0	+	0	0	+

Table 3: Effects of Monetizing Public Debt

[11] Formally, we derive from (20) $\partial i / \partial m < 0$.

[12] In the appendix we provide the whole set calculations from which the tables are extracted from. With a relatively low parameter value a , a defined as fraction of transaction liquidity to total income, we find an inflationary effect. Furthermore, we find that higher values of a lead to a deflationary scenario.

Again, this is however not true for the transition period, since the government can indeed spend more money, due to the temporary net-receipts from additional public debt. The respective extra costs are suffered by those who have saved in their first period and are now partly expropriated by inflation. Although, the devaluation of savings will end in the steady state, the onetime increase in the price level leaves the households who live in the transition period with a corresponding loss in wealth.

Summarizing, both the Austrians and the Keynesians are right; yes, printing money in order to decrease the interest rate does work. It does even work in the long run. But also yes, this is a fraud. The costs are imposed on the households who live in the transition period by allowing for temporary inflation or deflation. Therefore, this appears a highly questionable way of spurring growth, although in the end a new steady state with both a higher total income and a higher price level can be reached.

The view of some extreme Austrians asserting that the interest rate was solely determined by time preference in the long run is to be rejected. According to our model, a positive rate of time preference neither is necessary nor sufficient for a positive rate of interest. The household's time preference is one of several, both real and monetary, factors which are important to consider. Our results also question those who believe that the interest rate would instantly vanish if only the evil impatience of the households could be overcome or be compensated by some public measures.

3. *Can the interest rate become negative in the long run?*

Samuelson (1958) famously showed in his path-breaking OLG-model that the interest rate in a pure barter economy without the existence of durable goods can become negative. In Samuelson's model households live for three periods, resulting in three distinct generations that coexist at the same time; the young, the middle aged and the elderly. The young have no wealth and receive only a small income; therefore, they seek to increase consumption by taking credit. Hence, all their consumption is credit based. For instance, if they want to invest in a new firm, they have to take additional credit. On contrast, the generation in the middle of their life does not only pay back the previously granted credit, but also save a provision for their later retirement. Finally, in the last phase of their life, the elderly consume the whole of their wealth including received interest payments. Thus, in summary, in this three-period life cycle model the middle generation gives credit as a net saver for the respective young in order to receive interest from the latter when they are old.

Assume that population decreases at a certain rate. With every period there are less young people and more elder people per head of the middle-aged. Obviously, the interest rate should go down, because there are more savers and relatively less borrowers in each period. As Samuelson shows, it can even happen that the interest rate becomes negative. This finding holds at least for a pure barter economy where neither durable goods nor money is available for the storage of wealth of the middle generation. To illustrate the reasoning behind this finding, see the following scenario of the smallest possible barter economy. Robinson Crusoe ages and wants to provide for his elder days when he will no longer be able to work as intensely as before. If there is no Friday to whom he can grant credit, he will have to save some apples, although he knows that part of them will already have spoiled until the day he would have liked to consume some. In other words, Crusoe need to be well prepared to accept a negative interest rate on his savings, apples, in order to survive at all. This is basically the same with several generations, when savings tend to exceed capital demand even at a zero interest rate. Thus, the latter will also turn negative in a barter economy without durable goods.

The existence of stable money can prevent a negative interest rate. By choosing stable money, savers would always have a preferable alternative to lending their assets for less than nothing. Thereby, it creates the so-called lower zero bound to expansionary central bank policy. However, that would worsen the welfare situation. According to Samuelson's golden rule, the interest rate should always be equal to the growth rate of the respective economy. With a declining population, a negative interest rate would indeed be desirable to prevent dynamic inefficiency. Moreover, it is often argued that a negative interest rate would be necessary to overcome situations like the Keynesian liquidity trap. For instance, Silvio Gesell (1916) famously sought to generate it by levying a tax on holding cash. Even modern economists, like Mankiw (2009), favor similar measures in order to spur economic growth in times of crisis.

It is exciting to ask whether our model allows for a negative interest rate or not. First, some differences must be pointed out in comparison with Samuelson´s approach. In particular, we have explicitly regarded only two generations instead of three. However, one can easily assume, that there is indeed another, young generation (children) which is fed by the adult and, hence, included in the latter`s consumption, so this is a minor point. The more important difference to Samuelson's life cycle model is that we have implicitly taken account for inheritance. In our model, the young generation immediately receives all the firm's profits as their income and, hence, effectively owns them. Consequently, they do not have to take any credits for consumption purposes but, on the contrary, are net savers themselves in order to provide for their retirement. Only equity is handed down to the younger generation, while borrowed capital sensibly remains in the possession of the elderly. Thus, in our model, a decreasing population would generate fewer savers, but relatively more elderly people who seek to dis-save by selling bonds to the market or turning their liquidity holdings into consumption. Moreover, with an unaltered number and size of transmitted firms, total capital demand for the sake of real investment would be the same as before. Thus we should ceteris paribus expect an increasing rather than a decreasing interest rate in an ageing society. On the other hand, many other effects, coming for example from the labor market, which are not covered by our simple model, have to be taken into account. Nevertheless, our model severely queries the unquestioned believe in the saving glut hypothesis.

We find that the capital market interest rate on our model can become negative, if the central bank increases debt money by a sufficient amount. Formally, even with non-pecuniary return on holding liquidity $l > 0$, a sufficiently high m will ultimately yield a negative capital market interest rate, $i < 0$.[13] In this case, households would not only hold liquidity as their only asset, but even turn to net borrowers at the capital market, $K_H < 0$. This in turn implies that the central bank would then be the only supplier at the capital market. In such a scenario, the net rate of return on real capital K_F is also negative. That means that the gross return is insufficient to earn at least the depreciation. With regard to the economy's welfare situation, this is also a questionable outcome, despite the fact that pure firm profits remain positive in our model.

To some extent, this scenario is similar to the current situation of financial repression in the European Monetary Union. Financial repression means that the interest rate is below the inflation rate and, hence, implies a negative real interest, at least for cash-holders and holders of relatively liquid nominal assets. Sometimes it is argued that this cannot be blamed to the ECB, because the negative interest was caused by real reasons like in Samuelson's model. However, in the light of our model, this line of argument is not very convincing. First, our model puts considerable doubt on the saving glut hypothesis, as inheritance is recognized, as it should be. Second, our model shows that the central bank can influence the capital market interest rate to become negative even in the steady state, if only sufficiently strong - although temporary - inflation is allowed for. According to our model, the occurrence of a negative interest rate is impossible in the steady state without an extremely expansive monetary policy.

IV. Summary

The interest theories of Böhm-Bawerk and Keynes complement each other. As has been shown, both real and monetary factors determine the interest rate. Unless an extreme form of Ricardian equivalence applies the central bank can indeed permanently lower the interest rate to a voluntarily chosen or even negative level. In our model, this leads to an increase in both total output and the capital stock. However, this necessarily comes at the cost of inflation. Although the price will only temporarily change, this is sufficient to make such an expansive monetary policy quite questionable from a welfare economic point of view. In particular, some of those who live in the transition period are partly deprived of their wealth.

Furthermore, we have argued that Böhm-Bawerk's three causes for the existence of an interest rate are still valid and fully consistent with contemporary economic theory. On contrast, it is not possible to reduce it to just one reason like it is done in the intertemporal discounting theory. Even in a pure barter economy at least one

[13] However, as numerical simulations show, it can never happen that $i + l < 0$. The respective denominator in (13) and (14) remains always positive. At least, this is true as long we do not allow for a negative price level which would apparently do not make any sense.

additional factor, namely the decreasing marginal utility of consumption in any period, has to be taken into account. Hence, adverse opinions like in Herbener (2011) appear to be highly questionable in the light of our analysis.

Of course, we have to admit that our analysis has some limitations. In particular, the two-part utility function and the simple production function without labor, which was employed, might be a matter of debate. Moreover, there is neither a foreign sector nor a private financial sector in our model. Also, we only allowed for one single capital market interest rate. On the other hand, as was shown in van Suntum (2013), the incorporation of these refinements does not change anything in principle. For instance, when foreign bonds are only imperfect substitutes to domestic bonds, for instance, the central bank possesses still the power to manipulate the capital market interest rate at home.

Another limitation of our model is the restriction to a one period maturity of capital and only two periods of life for households. These simplifications have been mainly chosen in order to simplify calculations because, with allowing for more periods, compound interest would creep into the equations and the model would no longer be solvable other than by numerical methods. Indeed, it would be quiet interesting to have a corresponding more sophisticated model. In particular, one could analyze the transition periods in more detail. As long as only steady states are considered, however, this limitation appears of less, if any relevance.

So hopefully this paper could shed some light on the long-term determinants of the interest rate at least and will yield fruitful subsequent research on this fascinating and exciting issue.

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Appendix: Numerical example

		Steady State Scenarios (exogenaous changes in bold figures)					Steady State Scenarios (exogenous changes in bold figures)				
Exogenous Variables	**Symbol**	**basic**	**low i**	**negative i**	**public debt**	**monetized**	**basic**	**low i**	**negative i**	**public debt**	**monetized**
Coefficent for transaction liquidity LT	a	0,10	0,10	0,10	0,10	0,10	**0,80**	**0,80**	**0,80**	**0,80**	**0,80**
rate of time preference	θ	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50	0,50
initial quantity of money	$\bar{M}$	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00	100,00
elasticity of production	β	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10	0,10
debt money (share of GDP)	m	0,00	**0,09**	**0,20**	0,00	0,10	0,00	**0,09**	**0,20**	0,00	**0,10**
public debt (share of GDP)	g	0,00	0,00	0,00	**0,10**	0,10	0,00	0,00	0,00	**0,10**	**0,10**
elasticity of idle liquidity	l	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20	0,20
Endogenous Varaiables											
capital market interest rate	i*	0,09	0,01	-0,05	0,29	0,09	0,09	0,01	-0,05	0,29	0,09
real transaction liquidity	$L_T/p = aY$	0,08	0,08	0,08	0,08	0,08	0,61	0,62	0,62	0,60	0,61
productive capital (= investment)	KF = I	0,07	0,08	0,08	0,06	0,07	0,07	0,08	0,08	0,06	0,07
total output (GDP)	Y	0,7670	0,7737	0,7785	0,7527	0,7670	0,7670	0,7737	0,7785	0,7527	0,7670
real capital offer individuals	K_H	0,07	0,01	-0,07	0,13	0,07	0,07	0,01	-0,07	0,13	0,07
real capital demand government	K_G	0,00	0,00	0,00	0,08	0,08	0,00	0,00	0,00	0,08	0,08
real capital offer central bank	K_M	0,00	0,07	0,16	0,00	0,08	0,00	0,07	0,16	0,00	0,08
check: capital market in equilibrium? (=! 0)		*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*
real firm profits = income young individuals	$\prod_F = Y_{H1}$	0,69	0,70	0,70	0,68	0,69	0,69	0,70	0,70	0,68	0,69
real idle liquidity	L_S/p	0,16	0,22	0,31	0,09	0,16	0,16	0,22	0,31	0,09	0,16
price level	p	423,14	429,91	435,50	597,19	626,46	129,33	129,16	129,10	144,03	143,57
indivual`s total wealth	V = KH + LS/p	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23
individual`s consumption in 1. Period	C_1	0,46	0,46	0,47	0,45	0,46	0,46	0,46	0,47	0,45	0,46
individual`s savings	S_1	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23	0,23
check: V = S ? (=! 0)		0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00
avarage interest rate on savings	i_v	0,03	0,00	0,02	0,17	0,03	0,03	0,00	0,02	0,17	0,03
individual`s consumption in 2. Period	C_2	0,24	0,23	0,24	0,26	0,24	0,24	0,23	0,24	0,26	0,24
check: $S_1(1+i_v) - (V+K_H*i) = 0$?		*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*
check: $Y - K_F(1+i) - Y_{H1} = 0$?		*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*
government consumption	C_G = i*KM - iKG	0,00	0,00	-0,01	-0,02	0,00	0,00	0,00	-0,01	-0,02	0,00
check: Y - I + C1 + C2 + CG = 0?		*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*	*0,00*
central bank profit	$\prod_M = (1+i)K_M$	0,00	0,07	0,15	0,00	0,08	0,00	0,07	0,15	0,00	0,08
individual`s saving rate	S_1/Y_{H1}	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33	0,33
indivual´s consumption structure	C2/C1	0,51	0,50	0,51	0,59	0,51	0,51	0,50	0,51	0,59	0,51
Utility (flows only)	U_F	0,37	0,37	0,37	0,38	0,37	0,37	0,37	0,37	0,38	0,37
Utility (stocks only)	U_S	0,55	0,72		0,35	0,55	0,55	0,72		0,35	0,55
Utility total	$U = U_F + U_S$	0,92	1,09		0,72	0,92	0,92	1,09		0,72	0,92