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**A large-dimensional factor analysis of the Federal Reserve's
large-scale asset purchases**

By

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Abstract

This paper assesses the economy-wide effects of US unconventional monetary policy shocks. A precise identification of the unconventional monetary policy shocks is achieved by imposing zero and sign restrictions on a number of impulse responses from a large-dimensional dynamic factor model. In particular, an unconventional expansionary monetary policy shock is identified as a shock that increases the Federal Reserve's market share of US treasuries and mortgage-backed securities, and leads to an improvement in the real economy and improved credit conditions.

I find that an unconventional monetary policy shock significantly drives down the long-term interest rate spread and the credit spread, and improves both the financial market conditions and the commercial and industrial loans activity. Moreover, the impact on the real economy is significant.

The roughly \$2 trillion purchases of mortgage backed securities by the Federal Reserve Bank avoided a severe downturn according to estimates from a counterfactual analysis.

Keywords: Unconventional monetary policy, zero lower bound, large cross-sections, dynamic factor model, factor-augmented vector autoregression (FAVAR), Expectation-Maximization algorithm.

JEL classification: C32, C55, E43, E62, E68.

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1 Introduction

During 2007-2008 the Federal Reserve responded to the mounting global financial crisis with several large cuts in the federal funds rate. As a result, the federal funds target rate reached the zero lower bound (ZLB) of $0 - 0.25\%$ in December 2008, down from 5.25% in August 2007, and further stimulus from conventional monetary policy was therefore exhausted. However, unconventional monetary policy initiatives were already implemented during that period to alleviate financial market dysfunctioning, and other initiatives were announced to provide further stimulus to a deteriorating economy. These initiatives are often collectively termed quantitative easing (QE) and large-scale asset purchases (LSAP) without a precise distinction between the terms.¹ In fact, the various expansions of the LSAP program are often denoted by QE1, QE2, and QE3. The purpose of the US LSAP program is to improve credit conditions, and raise aggregate demand by exerting a downward pressure on long-term interest rates through massive purchases of primarily longer-term government bonds and agency MBS by the Fed. Reduced government bond yields and MBS yields should in turn reduce more risky bond yields and risk premiums, through portfolio effects and central bank signaling in general; see e.g. Joyce et al. (2012) for a discussion. Because of the LSAPs, the balance sheet of Federal Reserve has expanded to an unprecedented extent, and the effectiveness of this form of monetary policy on the real economy is still an open research question.

The size of the unconventional monetary policy initiative and the severity of most recent crisis and the is illustrated in Figure 1. The upper panel shows how the Fed's holdings of US treasury bonds, MBS and federal agency debt increased ninefold from the first QE announcement in November 28, 2008 and until 2014:Q3.² The long-term interest rate (10-year government bond yield) has generally drifted downwards, undoubtedly influenced by

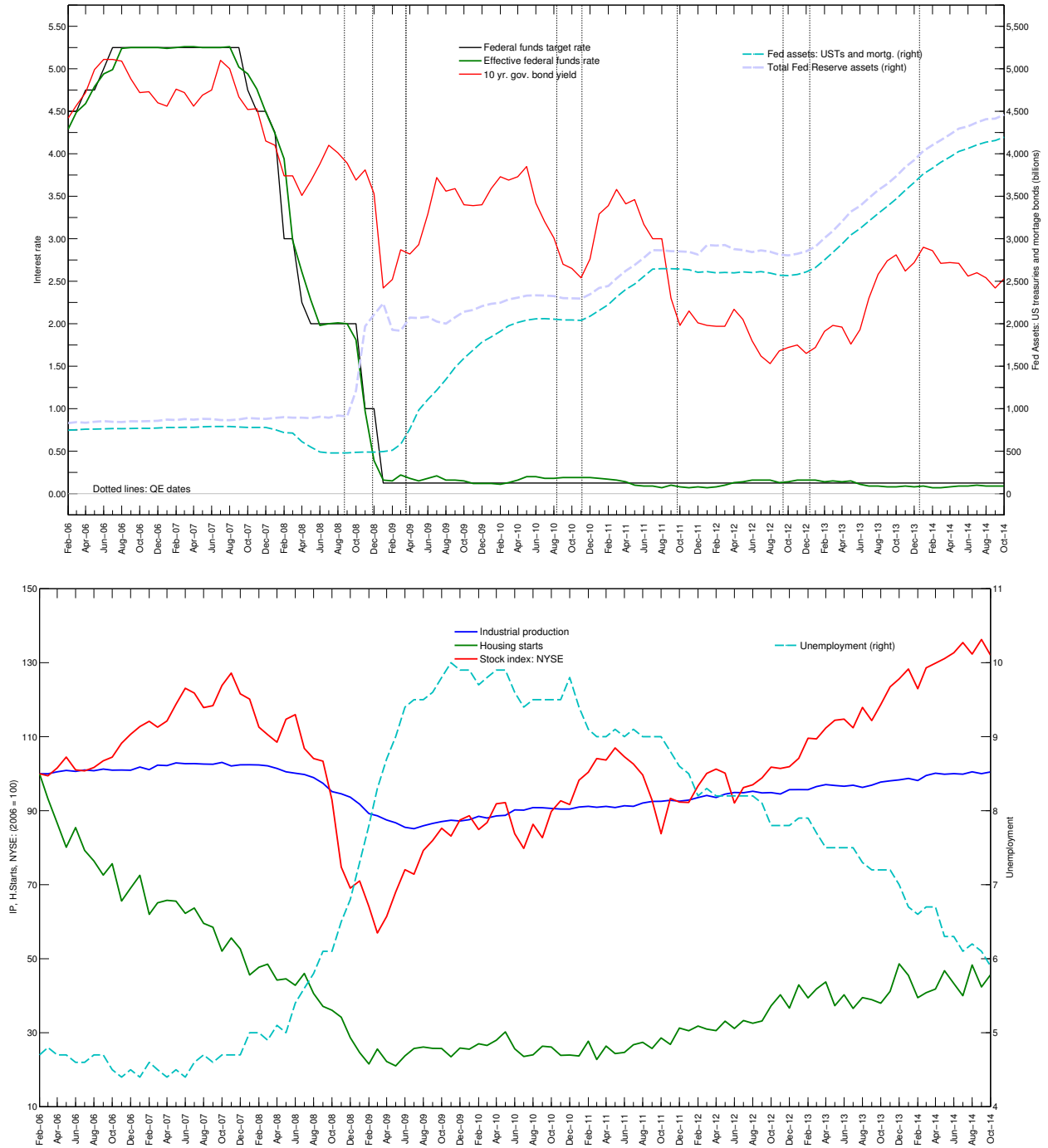
¹Some reserve QE to the policy targeted towards increasing the reserves of the commercial banks while LSAP is reserved to the policy targeted towards credit easing. The US and UK central bank are forerunners in LSAPs.

²Table C.1 contains an overview of the QE announcements.

the large cuts in the federal funds rate and the LSAPs. However, there are also periods where the long-term interest rate increases when the Fed actually expands the LSAPs or announces an expansion; see for instance the big increase in the 10-year yield after the 1,150 billion LSAP announcement on 18 March 2009, or a similar increase during the LSAP expansion from 12 December 2012.³ The lower panel of Figure 1 shows that the real economy has partly returned to the pre-crisis level after serious deterioration halfway into the period. Specifically, unemployment doubled the pre-crisis rate during October 2009 but is now approaching the pre-crisis rate. Moreover, industrial production has rebounded to its pre-crisis level; the stock market index is now above, while the housing market activity is still less than half of the pre-crisis level. Taking the two panels together, the conventional and unconventional monetary policy expansion seem to transmit to the long-term interest rate and real economy as expected, but while the empirical literature on the conventional monetary policy transmission mechanism is substantial, more empirical evidence is needed on the unconventional monetary policy.

A substantial part of the empirical literature on unconventional monetary policy focuses on the impact on the 10-year government bond yield or its spread to the federal funds rate. Furthermore, most of the structural analyses are in low dimensional VARs that typically include either the 10-year spread or the Federal Reserve's holdings of securities, but not both. One might worry, however, that four or five variables in a VAR is insufficient to uncover the unconventional policy shocks, especially if the Federal Reserve holdings are excluded. Furthermore, the LSAP is targeted towards an improvement of overall financing conditions, so a narrow focus on government bond yields with negligible credit risk and an exclusion of the more risky bond yields is probably not desirable in an assessment of the real effects of LSAP. In fact, during troubled times investors drive down bond yields when they seek safe haven in government bonds, with the risk that a negative innovation to the 10-year

³A partial explanation for this observation may be that the Fed's announcement of unchanged interest rates for an 'extended period' was more credible in the latter period as shown by Swanson & Williams (2014).



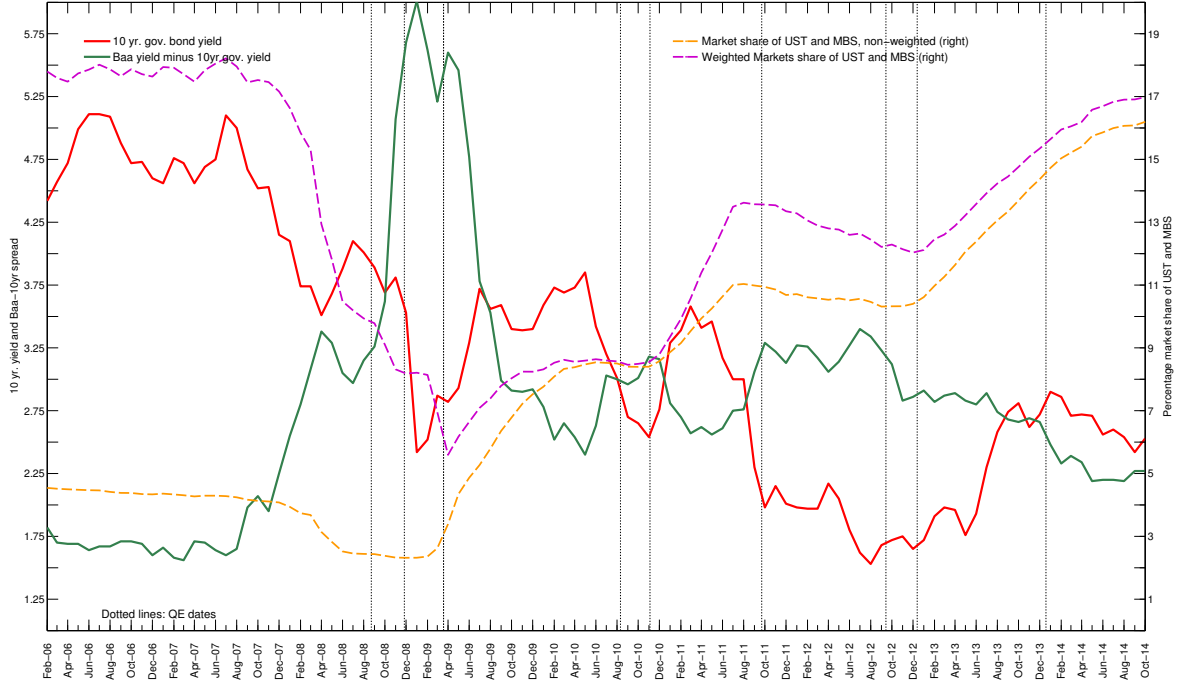
The upper panel shows the federal funds rate(s) and the 10-year government bond yield on the left scale. On the right scale is shown (i) the Federal Reserve Bank holdings of UST and MBS, and (ii) Total Federal Reserve Assets. The vertical lines show QE announcement dates: Aug. 22, 2008; Nov. 25, 2008; Mar. 18, 2009; Oct. 10, 2010; Nov. 3, 2010; Sep. 21, 2011; Sep. 13, 2012; Dec. 12, 2012; Dec 18, 2013. The lower panel shows total industrial production, housing starts and the New York Stock Exchange index on the left scale; all normalized to 2006 : 01 = 100. The right scale shows the unemployment rate.

Figure 1. Interest rates and key economic variables during the recent crisis.

bond yield (expansionary policy proxy) is associated with deteriorating fundamentals. The following observations from 2008-2009 may further highlight the challenges in estimating the effects of unconventional monetary policy.

Figure 2 shows, that during the fall of 2008, the 10-year government bond yield decreased, but the overall financing conditions continued to be depressed, as measured by a wide Baa credit spread. However, during the spring of 2009 the situation reversed with a widening of the 10-year term spread and a narrowing of the Baa credit spread. Interestingly, both periods involved QE announcements, but during the first period, with a decreasing term spread, the Fed's inventory of US treasuries (UST) and mortgage backed securities (MBS) was largely unchanged but its market share decreased, while during the second period with an increasing term spread, its market share increased, which is somewhat counter intuitive. However, the effect on the credit conditions seems to be as expected. Obviously, we need to replace eyeballing of these two periods with a model to sort out unconventional policy shocks, but the model should be able to pair innovations in the 10-year term spread with innovations in e.g. the Fed's assets and credit conditions in general.

This paper contributes to a growing literature on the real effects of unconventional monetary policy by estimating US economy-wide responses to such policy shock using a large-dimensional dynamic factor model. This model approach conditions the estimated policy response on a much larger information set and facilitates a structural sign identification based on more variables than in a standard VAR. Thus, in line with the discussion above, more data and new data are used to uncover the real impact of unconventional monetary policy compared to the existing literature. Furthermore, more variables are used to identify the policy shocks including the 10-year term spread but also real variables, credit spread, and new measures of the market impact of the Federal Reserve's LSAP. The empirical results suggests that an unconventional monetary policy shock leads to a widespread improvement in production, (un)employment, orders, housing, credit and financial conditions.



On the left axis is shown the 10-year government bond yield and the credit spread (Baa yield - 10yr gov. yield). On the right axis is shown (i) the Federal Reserve’s market share of US treasuries (UST) + mortgage backed securities (MBS), and (ii) a weighted market share of US treasuries and mortgage backed securities, respectively. The non-weighted market share simply measures the Federal Reserve’s holdings of UST and MBS relative to the total outstanding value. The weighted market share is weighted by the relative balance sheet size (B_i), as a fraction $B_{UST}/(B_{UST} + B_{MBS})$ of the UST market share and 1 minus this fraction of the market share of MBS.

Figure 2. 10-year government bond yield, credit spread, and LSAP activity.

The related literature on the effects of the recent unconventional monetary policy expansion can generally be organized into papers that analyze the short-term impact on long-term interest rates only, and papers that analyze the impact on the real economy over a longer horizon. Both directions are important, i.e. quantifying the policy impact on long-term interest rates is a useful first step in learning to what extent this policy is able to stimulate the real economy through the long-term interest rate.

Using event studies, Gagnon et al. (2011), Krishnamurthy & Vissing-Jorgensen (2011), and Glick & Leduc (2012) report cumulative QE1 announcement effects on the 10-year government bond yield of about -100 basis points (bps). The surprise component of the QE2 program of -18 bp. is significantly smaller in Krishnamurthy & Vissing-Jorgensen (2011), presumably because of a larger degree of financial market anticipation of the QE2

program. Note that these numbers do not measure long-run effects but rather cumulative effects over a set of one-day windows, including some announcement days with some positive effects on bond yields but with most effects being negative. Gagnon et al. (2011), D’Amico et al. (2012), and D’Amico & King (2013) find relatively smaller effects, around -40 to -50 bp., using time series regressions. These studies, however, cannot tell us whether the reported decrease in yields leads to an improvement in the real economy.

The QE impact on the real economy has been assessed in different types of structural VARs, including small time-varying parameter VARs (TVP-VAR), Markov-switching VARs (MS-VAR), large Bayesian VARs (LBVAR), and panel VARs (P-VAR). Counterfactual analyses and impulse responses are generally key ingredients in assessing the QE impact.

The large-scale BVAR of Lenza et al. (2010) and Kapetanios et al. (2012) and the dynamic factor model in this paper have their merits and demerits compared to the low-dimensional TVP-VAR in Baumeister & Benati (2013) or the MS-VAR in Kapetanios et al. (2012). Generally, it is computationally prohibitive to have time-varying parameters in the heavily parameterized large-scale models, so the TVP-VAR and MS-VAR have their merits if there are large structural breaks in the underlying variables or in the underlying structural relations. Therefore, I assume a reasonable stable structural relationship throughout the sample I consider. Note that Stock & Watson (2009) and Bates et al. (2013) find that factor models are rather robust to parameter instabilities. On the other hand, time-varying parameter models are usually low-dimensional and low-order due to the computational complexity, with the risk that the structural shocks cannot be uncovered (non-fundamentalness). Large-scale models, however, are less vulnerable to non-fundamentalness and deficient information sets because the factor estimates and structural shocks are based on a large information set.

With 4 and 6 variables, respectively, Baumeister & Benati (2013) and Kapetanios et al. (2012) identify an unconventional monetary policy shock as a shock to the 10-year term

spread but without reference to the Federal Reserve's assets.⁴ In contrast, Gambacorta et al. (2014), define an unconventional monetary policy shock as a shock to the central bank assets but without reference to the term spread in their P-VAR with 4 variables including the VIX index. A distinctive contributing feature of this paper is the use of more variables and new variables in a data-rich identification of unconventional policy shocks, which takes advantage of the dynamic factor model's large set of potentially useful identifying variables without necessarily expanding the dimension of the state variables. Specifically, in this paper an expansionary unconventional monetary shock is identified as an expansion in the Federal Reserve's market share of US treasuries and/or MBS that has a compressing market impact on credit spreads and a positive impact on credit conditions, financial market conditions, employment, and inflation. However, measures of financial market conditions, like financial condition indices or the VIX index⁵, do not need to be included among the state variables, but are merely used to achieve a sharper identification of the policy shock.

The empirical results of this paper generally support the modest empirical evidence on the real effects of unconventional monetary policy, but are much broader in the coverage of the US economy. Baumeister & Benati (2013) perform a counterfactual analysis of the real effects of a 10-year term spread that is 60 bps higher throughout 2009. The higher term spread would have resulted lowered inflation by 1 percentage point, lowered real GDP by 0.9%, and increased unemployment rate by 0.75 percentage point. Kapetanios et al. (2012) consider a TVP-VAR, MS-VAR and a LBVAR for the UK and use an identification approach similar to Baumeister & Benati (2013). Their counterfactual analysis with a 100 bp higher term spread is significantly model dependent, whereby the TVP-VAR and MS-VAR deliver more than a 5 percentage point decrease in GDP and more than a 3 percentage point decrease in inflation, while the LB-VAR results in 1 and 0.28 percentage

⁴It should be noted that Baumeister & Benati (2013) use the term "pure spread shock."

⁵Chicago Board Options Exchange Market Volatility Index

point, respectively. Gambacorta et al. (2014) consider a narrow sample from 2008:01-2011:06, but their panel VAR of eight countries should add up for the loss of power due to the small sample, according to the authors. Across the eight countries, the responses of prices and output to balance sheet shocks are generally similar and significantly positive. Wu & Xia (2014) replace the federal funds rate in a large dynamic factor model with a shadow rate, which can be negative when the federal funds rate is stuck at the ZLB. The shadow rate shock is identified recursively and the impulse responses are comparable to the evidence on conventional policy shocks, although a price puzzle seems to exist in Wu and Xia's (2014) analysis.

The remainder of this paper is organized as follows. The structural dynamic factor model (SDFM) is presented in Section 2; identification issues and the estimation method are presented in Section 3. Section 4 describes the data, Section 5 details the empirical results, and Section 6 concludes.

2 Model framework: A dynamic factor model

Since the first generation of dynamic factor models (DFM) of Geweke (1977) and Sargent & Sims (1977), a considerable amount of research has been devoted to the econometric theory and empirical analysis of approximate⁶ factor models of high dimension, notably the generalized dynamic factor model by Forni et al. (2000, 2004, 2005) and the static representation of the dynamic factor model by Stock & Watson (2002*a,b*). The premise of the dynamic factor model is that a set of N observed time series variables obey a factor structure, such that the comovement of the N variables can be described in terms of $q \ll N$ common dynamic latent factors, f , while the remaining unexplained portion of a variable resides in an idiosyncratic component ξ specific to each variable. As in Forni et al. (2005),

⁶In the first generation *exact* factor models like Ross (1976), Geweke (1977), Sargent & Sims (1977), and Geweke & Singleton (1981), the idiosyncratic components are orthogonal. However, the *approximate* factor models allow for some "local" correlation among the idiosyncratic components.

the time t observed variables in X_t are linear combinations of the current and lagged values of the dynamic factors:

$$X_t = \lambda^\top(L) f_t + \xi_t,$$

where $\lambda(L) = \lambda_0 + \lambda_1 L + \dots + \lambda_s L^s$ is a $q \times N$ polynomial matrix of factor loadings in the lag-operator L , and where the common factors dynamics are given by a $VAR(h)$ process

$$f_t = \phi(L) f_{t-1} + u_t$$

with $\phi(L) = \phi_1 L + \dots + \phi_h L^h$ being a $q \times q$ polynomial matrix of autoregressive coefficients. The dynamic factor model can be written in a state space form

$$\begin{aligned} X_t &= \Lambda F_t + \xi_t \\ F_t &= \Phi(L) F_{t-1} + U_t \end{aligned} \tag{1}$$

where the dimension of $F_t = [f_t^\top, \dots, f_{t-s}^\top]^\top$ is $r = q(s+1)$ and $\Lambda = [\lambda_0, \dots, \lambda_s]$ is $N \times r$. Moreover, $\Phi(L)$ is of order $p = \max(1, h-s)$ and therefore depends on the response heterogeneity of the panel (s) relative to h ; see Bai & Ng (2007) or Bai & Wang (2014b) for more details. Furthermore, $U_t = [u_t^\top, 0_{qs \times 1}^\top]^\top$. To fix ideas, I assume that $\xi_t \sim N(0, R)$ with R being a diagonal matrix corresponding to an exact factor model, but this assumption is later relaxed. Throughout the paper, I assume $u_t \sim N(0, W)$. The VAR component in (1) is assumed stationary and hence invertible. This implies that a vector moving average representation (VMA) of the model exists

$$X_t = C(L) u_t + \xi_t \tag{2}$$

where $C(L) = \Lambda [I - \Phi(L)^{-1}] V$, and V is a selection matrix such that $U_t = V u_t$.

The unknowns in this Gaussian state space model are the parameters in $\Theta = \{\Lambda, R, \Phi(L), Q\}$ and the latent dynamic factors f_t . However, the state space system in (1) is not economet-

rically identified as it is possible to form observationally equivalent models by arbitrary rotations of the latent factors, F_t , and the loadings Λ . Consequently, it is not possible to estimate a unique set of parameters $\hat{\Theta}$ with the data unless identifying restrictions are imposed on Θ . In addition, further restrictions are needed to identify the structural DFM from the reduced form DFM in (1). Section 2.1 discusses the econometric identification, Section 2.2 covers structural identification, and Section 3 describes maximum likelihood estimation.

2.1 Econometric identification

The dynamic factor model above is estimated with a fully parametric maximum likelihood method, the iterative Expectation Maximization (EM) algorithm, and this method is discussed shortly. A number of parametric identification approaches have been proposed in the literature. The predominant starting point is uncorrelated factors, which implies that the identification of the sources of variation in the panel X is then a matter of imposing an identifying structure on the loading matrix; in particular, a hierarchical structure that embodies the separation of the contribution of the factors to the variation in X . Important contributions describing this approach include the seminal paper by Geweke & Singleton (1981) (proposition I), Geweke & Zhou (1996), and Aguilar & West (2000). Alternatively, the assumption about uncorrelated factors can be relaxed by allowing for correlated factors. However, less restricted factor dynamics would have to be compensated for by a more restrictive simple structure on the loading matrix, in order to be able to separate the sources of variation; see Geweke & Singleton (1981) (proposition II) and recent work by Bai & Wang (2014*b*) and Bai & Wang (2014*a*).

In this paper, the identification scheme with correlated factors is preferred as most economic factors would be correlated, in contrast to the orthogonal factors from the popular principal components methods. Essentially, the identification approach for correlated factors constrains a small $q \times q$ subset of the large $N \times q(s + 1)$ loading matrix to be an

identity matrix. The variance-covariance matrix of the reduced form VAR residuals, W , is left completely unrestricted, which is ideal for the structural analysis.

Although the dynamic factors are identified by either of the above identification schemes, they are still unobserved and latent in nature. But macroeconomic data are generally prone to measurement errors, so a latent dynamic factor representation is a useful device to distinguish the underlying trend of inflation or employment, for example, from measurement errors.⁷ Yet, in some cases it makes more sense to analyze structural policy shocks to a real and observed series rather than a factor representation of a series; for instance, the perfectly measured federal funds rate. Consequently, I follow Bernanke et al. (2005) and augment the factors F_t with the perfectly measured federal funds rate, so that the factors are estimated jointly with this important variable.

2.2 Structural identification

The VMA representation of the DFM in (2) is not unique since the impulse responses are not yet structurally identified. A popular approach to the structural identification is the recursive identification by the computationally convenient Cholesky decomposition of $\text{Cov}(u_t)$, which would effectively impose $q(q+1)/2$ exactly identifying zero restrictions. Bernanke et al. (2005) impose a recursive identification scheme directly on the VAR impact matrix in their factor-augmented VAR (FAVAR) while Forni & Gambetti (2010) impose the recursive identification scheme on the impulse responses, that is, on $C(L)$. The Cholesky decomposition, however, imposes a causal chain on the shocks, and the imposition of a strict number of zero restrictions may lack economic motivation.

As an alternative to the exactly identifying zero restrictions, Faust (1998), Uhlig (2005), and Canova & Nicolò (2002) propose to set identify SVAR models by imposing sign restrictions on the impulse responses. Uhlig, for instance, proposes to identify monetary policy shocks by imposing only weak sign restrictions on the impulse responses over a given pe-

⁷Sargent (1989) shows how the existence of measurement error leads to a dynamic factor index model.

riod. In particular, he defines a contractionary monetary policy shock as one that leads to a negative price response, a positive response of nonborrowed reserves, and a non-negative response of the federal funds rate for a certain period following the shock. The idea is to generate a large number of alternative models using orthogonal rotations and to keep only the draws that generate impulse responses that are consistent with the economically motivated sign restrictions. This means that a unique model is not identified; instead, a set of admissible structural models that all are consistent with the sign restrictions is identified. Thus, a hypothesis can be ruled out if none of the draws satisfies the restrictions, or one can question whether the range of admissible impulse responses is too wide. More confidence in the identification of the structural shock will emerge if the shock satisfies a number of sign restrictions, if it seems distinctive from other competing shocks, and if the range of impulse responses is narrow and significantly different from zero; see Fry & Pagan (2011) and Kilian (2013) for a thorough discussion.

The starting point for generating candidate models is a base set of uncorrelated structural shocks and this basis is then rotated into a new model with a new set of uncorrelated shocks and impulse responses. The Cholesky decomposition of $\text{Cov}(u_t) = A_0^{-1} (A_0^{-1})^\top$ is a simple way to have uncorrelated structural shocks given by $\varepsilon_t = A_0 u_t$. Rotating ε_t by the orthogonal matrix Q gives a new set of uncorrelated shocks, $\tilde{\varepsilon}_t = Q^\top A_0 u_t$, where $Q^\top A_0$ in general will be nonrecursive. The new impulse responses are then given by $\tilde{B}(L) = B(L)Q = C(L)A_0^{-1}Q = \Lambda [I - \Phi(L)^{-1}] V A_0^{-1}Q$, which can be evaluated against the imposed restrictions.

Sometimes the impulse responses are required to satisfy a combination of zero and sign restrictions, for instance in separating monetary policy shocks from aggregate demand shocks. This could be accomplished by requiring that real variables do not respond within the period to a monetary policy shock, i.e. a zero restriction. However, the combination of zero restrictions with sign restrictions has been quite difficult to implement in practice, although a penalty function approach has been proposed by Mountford & Uhlig (2009).

Recently, Arias et al. (2014) propose a fast algorithm that draws Q from the correct distribution of sign restrictions conditional on the zero restrictions.⁸ They show that the existing algorithms implicitly introduce sign restrictions in addition to the ones specified in the identification, which generate biased impulse response functions and artificially narrow confidence intervals.

The structural identification in this paper relies on a combination of zero and sign restrictions and builds on the algorithm of Arias et al. (2014). The appendix contains a summary of the algorithm. In the light of the LSAP program, I define and focus on an unconventional monetary policy shock but also define an expansionary conventional monetary policy shock, an aggregate demand shock, and aggregate supply shock; the latter three shocks are helpful in addressing the "multiple shock problem" discussed in Fry & Pagan (2011). Specifically, I define an unconventional monetary policy shock (uMP) as one that (i) increases the Federal Reserve's market share of USTs and MBS, (ii) decreases the credit spread, (iii) improves the financial market conditions, and (iv) increases inflation and output (employment, industrial production). An expansionary conventional monetary policy shock (cMP) is defined as a decrease in the federal funds rate and an increase in inflation and output. To further distinguish the unconventional policy shock from the conventional, zero restrictions are imposed so that the federal funds rate does not respond initially to an unconventional policy shock. A positive aggregate supply shock (AS) is defined as a shock that leads to a decrease in inflation and an increase in output; a similar definition is seen in Baumeister & Benati (2013). A positive aggregate demand shock (AD) moves inflation, output, and the interest rate in the same direction, which can be distinguished from a conventional expansionary monetary policy shock that would have the opposite sign on the interest rate.⁹ A summary of the shocks for my baseline model is

⁸See also Binning (2013) for a related paper.

⁹To further separate policy shocks from non-policy shocks, one could impose zero restrictions on the initial response of inflation and output to policy shocks. The results are not found to be sensitive to imposing these zero restrictions.

given in Table 1, where (+) or (−) indicate the required sign, (*) means unrestricted, and (0) indicates an initial zero restriction. In addition, one may define the sign restriction to hold in each of J_i periods or cumulatively over the J_i periods, while the zero restrictions may be required to hold for one or more periods.

Table 1. Sign restrictions for the baseline dynamic factor model

		<i>Shocks:</i>			
		uMP	cMP	AS	AD
	<i>Series:</i>				
Aggregate inflation:	CPI-U all	+	+	−	+
Aggregate output:	IP: total	+	+	+	+
	Empl: total	+	+	+	+
	Unemp: All. [†]	*	*	*	*
Unconventional policy	Mkt. share	+	*	*	*
	FRB assets	+	*	*	*
Conventional policy	Fed funds	0	−	*	+
Financial conditions	Vix	−	*	*	*
Credit conditions	Baa - 10y	−	*	*	*

Note: The first column contains the theoretical economic concepts that the structural shocks should affect. The second column shows the restricted observed series; see variable numbers 106, 2, 29, 23, 98, 71, 120, and 88 in Appendix A. The last three columns show the shocks and the identifying restrictions. [†]I want to remain agnostic about this variable.

3 Estimation

The linear Gaussian state space model in (1) with its latent factors f_t is well represented in a Kalman filter setting. Building on the seminal work by Dempster et al. (1977), Shumway & Stoffer (1982), and Watson & Engle (1983) introduce the EM algorithm to estimate the parameters in state space models as in the model above. Doz et al. (2012) also use the EM algorithm in their study of the asymptotic properties of QML estimation for large approximative factor models and show that this method is robust to misspecification that arises from weak cross-sectional and serial correlation of the idiosyncratic errors. Finally,

Jungbacker & Koopman (2014) show how one could speed up the maximum likelihood estimation of a large DFM.

Essentially, the EM algorithm is an iterative maximum likelihood method that switches between an Expectation step and a Maximization step. The maximization step results in the following closed form estimators at iteration j

$$\Lambda^{(j)} = DC^{-1} \quad (3)$$

$$R^{(j)} = \frac{1}{T} \sum_{t=1}^T \left(X_t - \Lambda^{(j-1)} \hat{F}_{t|T} \right) \left(X_t - \Lambda^{(j-1)} \hat{F}_{t|T} \right)^\top + \Lambda^{(j-1)} \hat{P}_{t|T} [\Lambda^{(j-1)}]^\top \quad (4)$$

$$\Phi^{(j)} = BA^{-1} \quad (5)$$

$$W^{(j)} = \frac{1}{T} [C - BA^{-1}B^\top] \quad (6)$$

where the following moments are available from the Kalman smoother (indicated by subscript $t|T$):

$$\begin{aligned} A &= \sum_{t=1}^T \left(\hat{F}_{t-1|T} \hat{F}_{t-1|T}^\top + \hat{P}_{t-1|T} \right) & B &= \sum_{t=1}^T \left(\hat{F}_{t|T} \hat{F}_{t-1|T}^\top + \hat{P}_{\{t,t-1\}|T} \right) \\ C &= \sum_{t=1}^T \left(\hat{F}_{t|T} \hat{F}_{t|T}^\top + \hat{P}_{t|T} \right) & D &= \sum_{t=1}^T X_t \hat{F}_{t|T}^\top \end{aligned} \quad (7)$$

and where $\hat{F}_{t|T} = E[F_t | \mathbb{X}_T]$, $\hat{P}_{t|T} = \text{var}(F_t | \mathbb{X}_T)$, $\hat{P}_{\{t,t-1\}|T} = \text{Cov}(F_t, F_{t-1} | \mathbb{X}_T)$ and $X_T = \{X_1, \dots, X_T\}$ denotes the information set. The estimated parameters from iteration j in $\Theta^{(j)} = \{\Lambda^{(j)}, R^{(j)}, \Phi^{(j)}, W^{(j)}\}$ in addition to certain initial values, can then be used in the expectation step to compute a new set of moments from the Kalman smoother. Subsequently, the estimated moments are supplied to the maximization step above from which $\Theta^{(j+1)}$ can be calculated, and the procedure continues until convergence of the likelihood. The econometrically identifying restrictions discussed in Section 2.1 are not yet imposed on the estimated loadings in $\Lambda^{(j)}$ because these are still fully unrestricted. However, Bork et al. (2009) derive the loading estimator subject to a set of linear loading restrictions in

the form $H_\Lambda \text{vec } \Lambda = \kappa_\Lambda$ that takes this form

$$\begin{aligned} \text{vec } (\Lambda^{(j-1)*}) &= \text{vec } (\Lambda^{(j)}) \\ &+ (C^{-1} \otimes R^{(j-1)}) H_\Lambda^\top [H_\Lambda (C^{-1} \otimes R^{(j-1)}) H_\Lambda^\top]^{-1} \times \\ &+ \{ \kappa_\Lambda - H_\Lambda \text{vec } (\Lambda^{(j)}) \} \end{aligned} \quad (8)$$

The Kalman filter and the EM algorithm can also handle missing observations among the observed variables. A specific form of missing observations can be seen in unbalanced panels in which some of the included time series may have their first observation later in the sample compared to other series with a full sample. In this paper, I want to condition the policy response on measures of financial market conditions, but such measures typically have their first observation in the 1970s or later.¹⁰ Nevertheless, it is possible to filter the series with missing observations based on their estimated loadings on the dynamic factors. It turns out that the required modification amounts to slightly changed estimators of (3), (4), and (8) only; see Shumway & Stoffer (1982) and Shumway (2000) for more details. In particular, I define a time-varying indicator matrix, I_t that has ones along the diagonal unless the i th variable has a missing observation at time t , in which case element (i, i) is zero. Then (3) and (4) becomes

$$\Lambda^{(j)} = \left[\sum_{t=1}^T \tilde{X}_t \hat{F}_{t|T}^\top + (I - \mathcal{I}_t) \tilde{\Lambda}_t^{(j-1)} A \right] C^{-1} \quad (3')$$

$$\begin{aligned} R^{(j)} &= \frac{1}{T} \sum_{t=1}^T \left\{ \left(\tilde{X}_t - \tilde{\Lambda}_t^{(j-1)} \hat{F}_{t|T} \right) \left(\tilde{X}_t - \tilde{\Lambda}_t^{(j-1)} \hat{F}_{t|T} \right)^\top \right. \\ &\quad \left. + \tilde{\Lambda}_t^{(j-1)} \hat{P}_{t|T} \left[\tilde{\Lambda}_t^{(j-1)} \right]^\top + (I - \mathcal{I}_t) R^{(j-1)} \right\} \end{aligned} \quad (4')$$

¹⁰To mention a few, the VIX option implied volatility series starts in 1986:06, and the Chicago NCFI condition index starts in 1973:01. For the other series, see the last 7 series in Appendix A.

where $\tilde{X}_t = I_t X_t$ is the time t observed variables that may have zero elements in case of missing observations and similarly for the rows in $\tilde{\Lambda}_t^{(j-1)} = I_t \Lambda^{(j-1)}$. Equation (8) should be based on $\Lambda^{(j)}$ in (3') and $R^{(j)}$ in (4').

4 Data

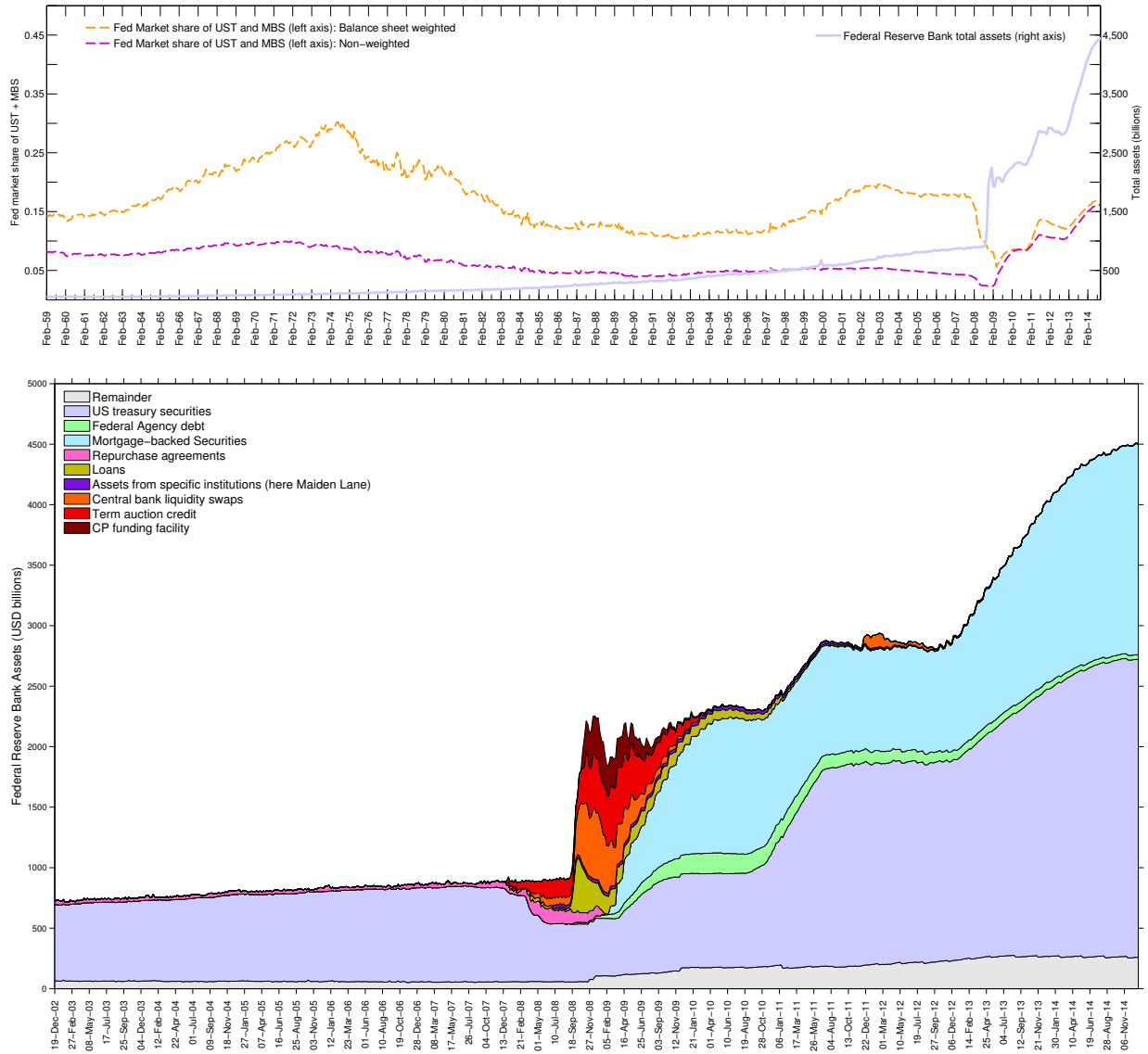
In general, the dataset is an updated and extended version of the balanced panel used by Bernanke et al. (2005). Updating their data to 2014:09 recovers 93% of their series as some series have been discontinued. Fourteen new variables are added to the extended panel, such that a total of 126 variables are included that cover 1959:01 to 2014:09. The new series are primarily measures of the Federal Reserve assets and its market share of UST and MBS. Important financial market condition measures are added to the panel in order to closely approximate the information that the Federal Reserve Board bases its decision upon. In particular, the National Financial Condition Index (NFCI) of the Federal Reserve Bank of Chicago is included, as well as the Kansas City Financial Stress Index, the VIX index, the TED spread, and the MOVE index, to mention a few of these series. These series, however, have a shorter sample so that the panel becomes unbalanced, but the EM algorithm can handle this as shown above. All the series are seen in Appendix A and the series in general represent the following categories of macroeconomic and financial time series: output and income; (un)employment, hours and earnings; housing; consumption, orders and inventories; money and credit; bond and exchange rates; consumer, producer prices, and commodity prices; stock prices; the Federal Reserve's assets, and financial condition measures.

The components of the Federal Reserve's balance sheet are in general not available online for the complete sample and have thus been obtained from various issues of the digitized Federal Reserve Bulletin archived at the Federal Reserve Archive (FRASER). As a result, the Federal Reserve's total assets and its holdings of UST and MBS have been obtained

partly from the Federal Reserve Bulletins and the H.4.1 releases at www.federalreserve.gov.

The total Federal Reserve Bank assets together with two calculated measures of the Fed's market share of UST and MBS are shown in Figure 2, with a recent decomposition of the assets into main components is shown in Figure 3. The motivation for including the two measures of the Fed's market share is that I want a measure of financial market impact of the LSAPs. Ideally, the market share of Treasury notes and bonds would be preferred as part of the unconventional monetary policy analysis, but a decomposition of the Federal Reserve Bank holdings of USTs into bills, certificates, and notes/bonds reveals significant switches from bills and certificates into notes and bonds, and from bonds and notes into bills and certificates, especially during 1959 and during the 1960s. Thus, the market share would sometimes fluctuate widely despite The Fed's largely unchanged balance sheet, so for this reason I work with an overall measure of the Fed's holdings USTs relative to the total amount outstanding of USTs. However, since 2009 the Fed has increased its holdings of MBS from zero to roughly 2,000 billion, resulting in a MBS inventory almost as large as its UST inventory. Thus, the market impact now clearly surpasses the US treasury market. Consequently, the simple UST market share needs to be redefined in terms of including MBS and I therefore consider two measures of market impact. Firstly, a non-weighted market share that simply measures the Federal Reserve's holdings of USTs and MBS relative to the total outstanding value of USTs and MBS. Secondly, I also consider a balance sheet weighted market share that is weighted by the relative balance sheet size (B), as a fraction $B_{UST}/(B_{UST} + B_{MBS})$ of the UST market share and 1 minus this fraction of the market share of MBS. This measure tracks the US treasury market impact up to 2009, and from then on it gradually includes the MBS market impact.¹¹

¹¹Although this measure is a specific weighting of market impacts, it may have the potential to reflect the financial markets' evolving perception of the Fed's intentions of lowering interest rates beyond the UST market.



The upper panel shows the monthly total value of the assets on the balance sheet of the Federal Reserve in a longer perspective (right axis). On the left axis is shown (i) the Federal Reserve's market share of US treasuries (UST) + mortgage backed securities (MBS), and (ii) a weighted market share of US treasuries and mortgage backed securities, respectively. The non-weighted market share simply measures the Federal Reserve's holdings of UST and MBS relative to the total outstanding value. The weighted market share is weighted by the relative balance sheet size (B_i), as a fraction $B_{UST}/(B_{UST} + B_{MBS})$ of the UST market share and 1 minus this fraction of the market share of MBS. The lower panel shows a weekly decomposition of the balance sheet into its main asset components since December 2002. Data sources: Federal Reserve Bulletin and H.4.1 releases at www.federalreserve.gov.

Figure 3. Total Federal Reserve assets and main asset components.

5 Empirical results

In the end, I estimate a baseline model with 6 dynamic factors from the panel of 126 time series and show that an expansionary unconventional monetary policy shock leads to a significant increase in industrial production, employment, inflation, housing starts, and capacity utilization, in addition to a decrease in the 10-year yield spread, the credit spread, and the market volatility. The background for these results are organized as follows. I first comment on the factor estimates and provide a brief discussion of the estimation procedure. Economy-wide impulse responses following an unconventional monetary policy shock are then presented with a counterfactual analysis. Finally, I present some robustness results and close this section with a discussion.

5.1 Dynamic factor estimates and the estimation procedure

The baseline model contains 6 dynamic factors that are related to inflation, unemployment, employment, the changes in the Federal Reserve’s market share of UST and MBS, the federal funds rate and the credit spread. The federal funds rate and the Federal Reserve’s market share are measured without error in (1). I choose to measure these two policy variables without error so that any estimated policy shock belongs to a precisely measured policy variable and therefore is uncontaminated by any factor approximation error of the policy variable.

The number of factors are determined by the information criterion by Hallin & Liska (2007), and their $IC_{1;n}^T$ and $IC_{2;n}^T$ point toward 4 – 6 factors. The information criterion has guide me in the number of factors but subsequently I have to decide on the $q = 6$ observed variables in X that should carry the identifying loading restrictions discussed in Section 2.1. A preliminary principal component analysis tell me what kind of variables will predominantly load on the first principal component, the second, etc., and this insight is used to determine the subset of X that have a simple loading structure. If the final

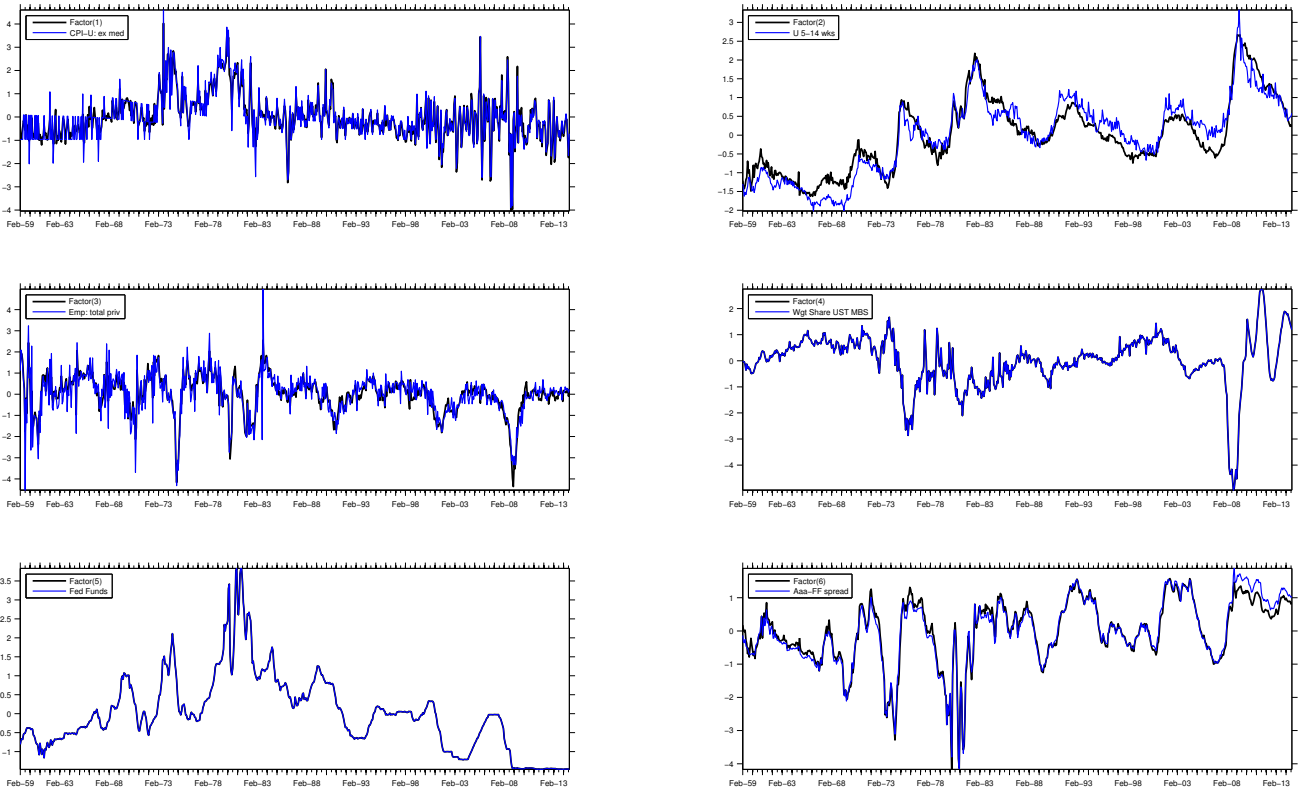
factor estimate turns out to be quite different from the characteristics of the restricted variable, then one may consider adjusting the identifying restrictions. However, it is my experience that the dominant factors are the ones listed in the beginning of this section. The number of lags in the VAR is $p = 12$, and this has been determined to be the most parsimonious model consistent with absence of residual autocorrelation, which is important for the structural analysis.

In the EM algorithm, I impose the exactly identifying restrictions on the inferred dominant factors and initially filter the factors with very weak priors on the initial parameter estimates. In particular, the loading matrix is filled with zeros except for the exactly identifying unit restrictions, and a few initial iterations with this simple loading matrix are undertaken in order to filter the factors. After the initial iterations, the complete loading matrix is estimated and the algorithm continues until the likelihood value and the parameter estimates have converged.

Figure 4 shows the estimated factors together with the single most correlated observed variable in the panel. The first factor captures the underlying inflation and the second factor is related to unemployment, whereby the latter has a 0.94 correlation with unemployment duration measures and a 0.85 correlation with the overall unemployment rate. The third factor is mainly related to employment growth but is highly correlated with measures of economic activity such as industrial production series (~ 0.75), capacity utilization (0.5), and PMI (0.82). The fourth factor is the Fed's market share of UST and MBS, the fifth is the federal funds rate, and the sixth factor is related to credit spreads and term spreads in general.

5.2 Impulse response analysis

Consider now the impulse responses for the baseline model. Based on the sign restrictions in Section 2.2, the results of the estimated economy-wide responses of an expansionary unconventional monetary policy shock are shown in Figure 5. In particular, an uncon-



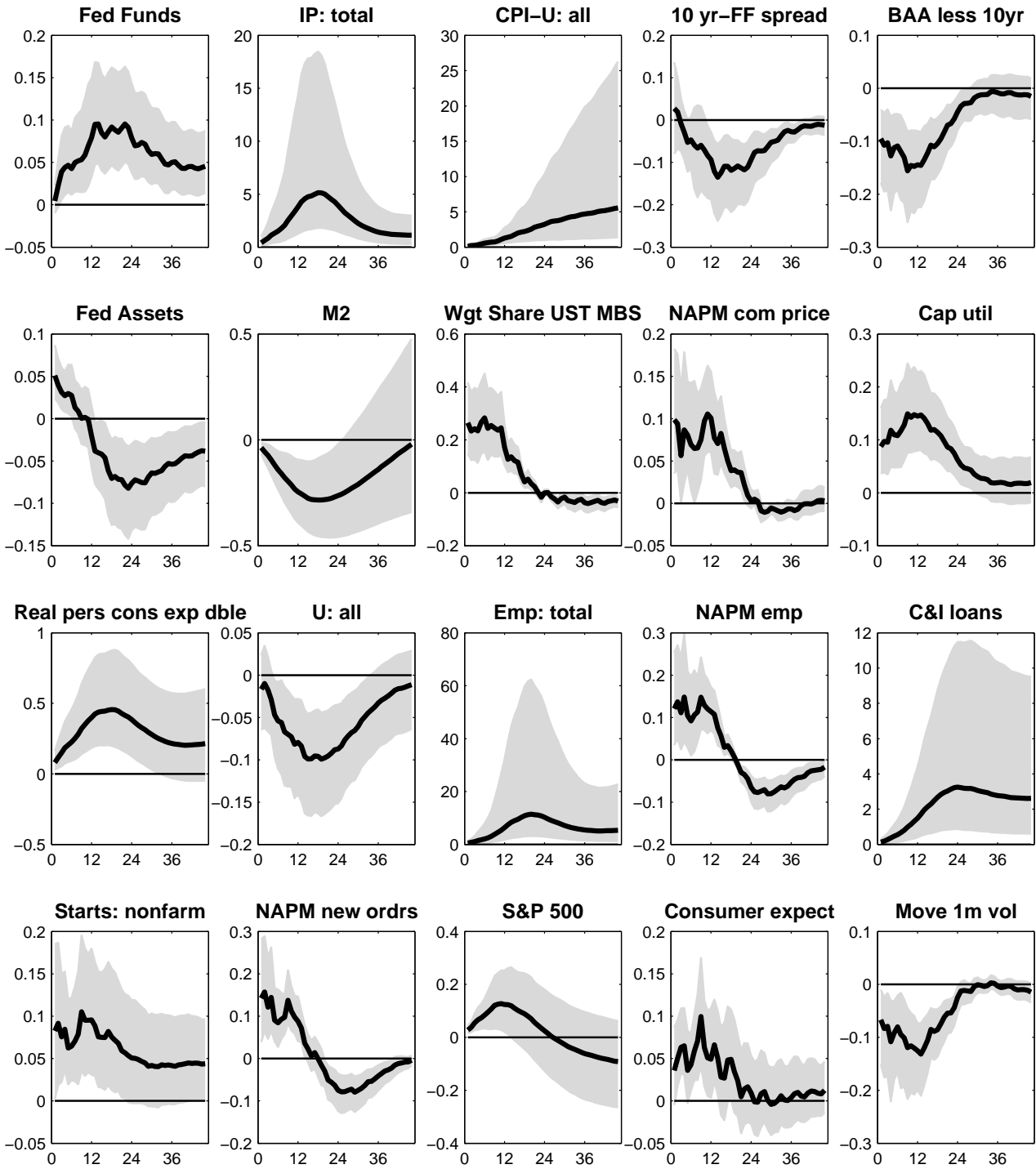
The figure illustrates the estimated factors from the baseline model. The legends show the observed variable in the panel with which the factor is most correlated with. CPI-U ex. med is consumer price inflation excluding the medicine component. U 5-14 is duration of unemployment. Emp. total priv is total private employment (in log differences). Wgt Share UST MBS is a balance sheet weighted market share of USTs and MBS by the Federal Reserve (in differences). This factor and the Fed Funds is perfectly measured. Aaa - FF spread is the spread between the AAA corporate bond yield and the federal funds rate.

Figure 4. Estimated factors in the baseline model.

ventional shock significantly drives down the long interest rate spread (10yr - FF spread); the credit spread (Baa less 10yr); and increases the commercial and industrial loans (C&I loans). Thus, the credit conditions seem to be improved. Industrial production, capacity utilization, inflation (CPI-U), and employment respond in a significantly positive way, while unemployment is significantly reduced. Note that I remain agnostic about unemployment as no sign restrictions are imposed. Financial market conditions seem to be improved as measured by a lowered MOVE volatility index. All impulse responses are normalized to show the response of a 1% innovation in the market share. For the included 25 variables, the responses are in general plausible and significant, so the LSAP program seems to have the desired impact on both credit conditions and the real economy. It should be noted that the positive response of the Federal Reserve's assets (measured in annual growth) is replaced by a negative response after one year, but the dynamics of this variable is indeed very special as seen in Figure 3. For other model specifications the longer-term response of this variable turns out to be insignificantly different from zero. The illustrated responses are robust to other model specifications and the choice of lags in the VAR as detailed in the section below with robustness analyses.

5.3 Counterfactual analysis

What would have happened if the Federal Reserve did not embark on LSAPs? Would the unemployment rate be much higher and would we see a worsening of the deflationary tendencies? Were the credit spread even higher? These are all important questions that we wish we could answer, in particular in evaluating the observed policy actions. In an attempt to provide some answers to these important questions, I perform a counterfactual analysis on the basis of my estimated baseline model. Specifically, I consider the hypothetical case of no Federal Reserve Bank purchases of MBS and analyze the counterfactual outcome in terms of (un)employment, inflation, output, credit spreads, and financial conditions. The key to this analysis is the elimination of the all the purchases of MBS resulting in a



The figure illustrates the impulse responses of key macroeconomic variables to a positive shock to unconventional monetary policy identified by **zero and sign restrictions**. All responses are normalized by considering a one percent change in the innovations to the market share (Wgt Share). Vertical axes are measured in standard deviations. Horizontal axes show time horizon. The 68% confidence intervals are shaded and based on the first 10^4 satisfied draws out of a total of 10^6 generated draws.

Figure 5. Impulse responses to an unconventional monetary policy shock: *Sign restrictions*.

counterfactual lower market share.¹² This approach focuses on the real effect of the LSAPs of MBS and is an alternative to the counterfactual analysis in Baumeister & Benati (2013) and Kapetanios et al. (2012) where 60 to 100 bps are added to the term spread.

Counterfactual analyses in SVARs are seen in a number of papers, including Bernanke, Gertler, and Watson (1997) and Herrera & Hamilton (2004). Imagine that we want to study the result of a counterfactual development in the j th variable in X_t during the period t to t^* , that is, to consider a particular counterfactual sequence $\{X_{j,\tau}^*\}_{\tau=t}^{t^*}$, where $X_{j,\tau}^*$ is different from the observed $X_{j,\tau}$. This amounts to choosing the structural shocks $\{\varepsilon_{j,\tau}^*\}_{\tau=t}^{t^*}$ such that $X_{j,\tau}^*$ is achieved, while the remaining shocks are unchanged.

The starting point for the counterfactual analysis is the historical decomposition of X_t in terms of the structural shocks. Because the market share has a simple loading structure (a single unit loading) it suffices to briefly describe the theory in terms of the moving average representation and the historical decomposition of the factors only.¹³ The structural moving average representation $F_t = \sum_{i=1}^{\infty} \Psi_j \varepsilon_{t-i}$ follows from (1) where the recursion for Ψ_j is provided in e.g. Luetkepohl (2011) or Lütkepohl (2005), and from this the historical decomposition can be derived as

$$F_t = \sum_{i=0}^{t-1} \Psi_i \varepsilon_{t-i} + \Phi_1^{(t)} F_0 + \dots + \Phi_p^{(t)} F_{-p+1} \quad (9)$$

with $[\Phi_1^{(t)}, \dots, \Phi_p^{(t)}]$ defined as the first r rows of the corresponding $rp \times rp$ companion matrix raised to the power of t . In case we seek a particular value of the $F_{j,\tau}^*$, for instance a counterfactual market share, this can be accomplished by choosing the structural shocks $\varepsilon_{j,\tau}^*$ such that the following holds

$$\sum_{k=1}^K \Psi_{jk,0} \varepsilon_{k,\tau}^* = F_{j,\tau}^* - \hat{F}_j \quad (10)$$

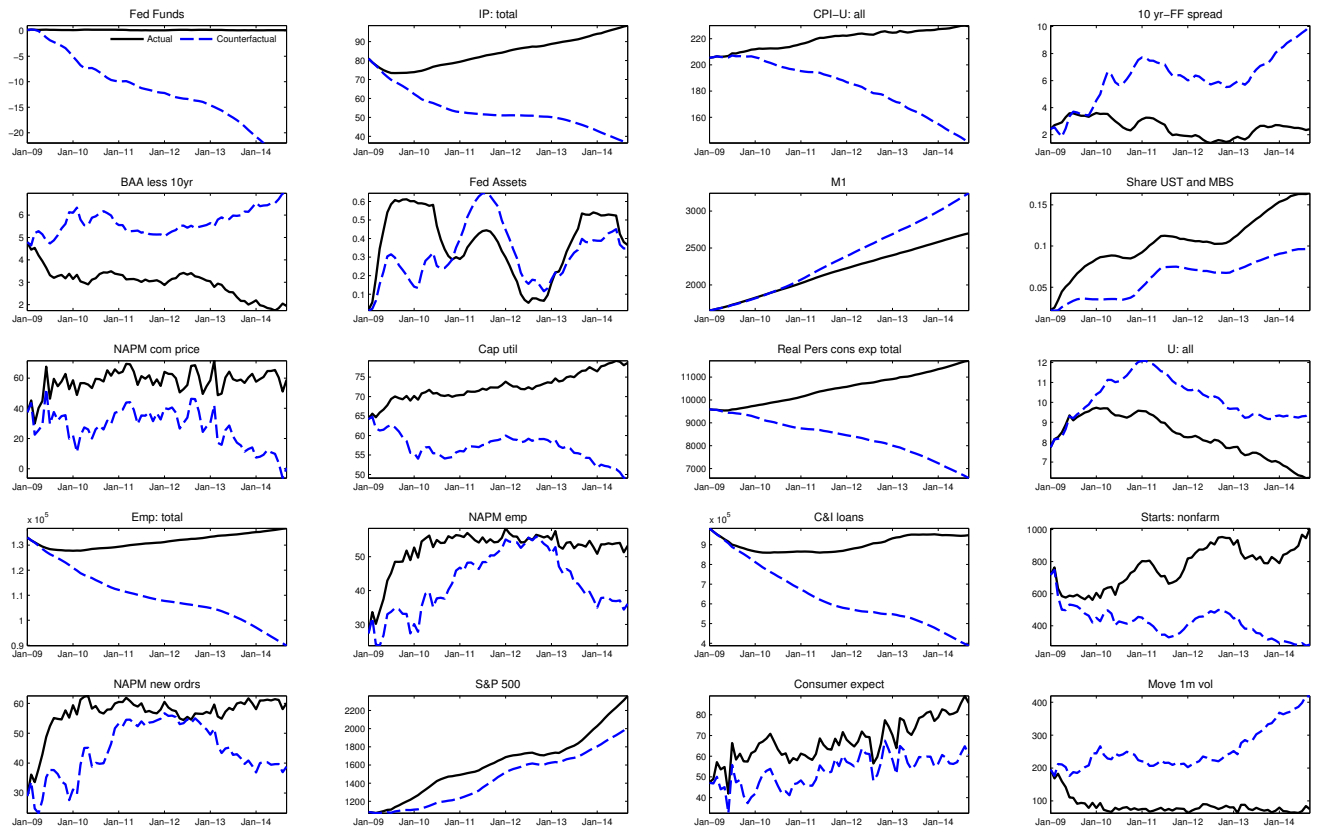
¹²Place a zero in front of MBS_{FED} in $(UST_{FED} + MBS_{FED}) / (UST_{Market} + MBS_{Market})$

¹³The counterfactual X_t follows from multiplying the counterfactual factors by the loadings.

where \hat{F}_j is the j th row of $\Phi_1^{(1)}F_0 + \dots + \Phi_p^{(1)}F_{-p+1}$.

Figure 6 shows the results of the counterfactual market share analysis beginning in January 2009 where the first purchase of MBS was recorded on the Federal Reserve balance sheet. Eliminating these purchases would result in a counterfactual six percent lower market share in September 2014 compared to the actual market share of 16%. The strong response of the federal funds rate will be discussed and dealt with in a moment, but notice that the absence of MBS purchases are estimated to result in lower production, lower inflation, higher term spread, higher credit spread, higher unemployment, lower employment, lower capacity utilization, higher financial market volatility, and depressed lending. In short: the LSAPs of almost 2 trillion MBS seem to help avoiding the disastrous outcome of deflation, financial distress, and a much more depressed economy. Now, the response of the federal funds rate to the absence of MBS purchases looks peculiar and a comment and a robustness analysis is needed. Firstly, nothing in the model prevents the federal funds rate to be negative and given the really bad state of the economy as seen by virtually all the key indicators in Figure 6 it is not surprising that the federal funds rate decreases. As a robustness analysis, however, I shut down the response of the federal funds rate to the counterfactual unconventional policy shocks in $\varepsilon_{j,\tau}^*$ by setting the appropriate $\Psi_{jk,0}$ in (10) equal to zero during the 2009-2014 period. This rules out most of the negative response of the federal funds rate but without changing the conclusions. Hence, the negative federal funds rate is not the main driver of the results.¹⁴

¹⁴Results are available on request.



The figure illustrates the results of a counterfactual analysis of no MBS purchases by the Federal Reserve Bank. This implies a lower market share of UST and MBS which is the key to the counterfactual analysis. All the results are based on the closest-to-median model; the so-called Median Target model of Fry and Pagan (2011).

Figure 6. Counterfactual analysis of a lower market share of UST and MBS.

The results are comparable with the results of Baumeister & Benati (2013). Although they primarily consider a counterfactual analysis for 2009 only, their counterfactual unemployment rate of 10-11% at the end of 2009 is comparable to the counterfactual unemployment rate in this paper while their deflation during 2009 appears a bit later here. In addition, the dynamic factor model approach in this paper allows to study many more counterfactual responses and may thus serve as further diagnostic checks compared to the low dimensional VAR models.

A concern in constructing any counterfactual analyses is the Lucas critique and the risk that agents may change their behavior. Note, however that the Federal Reserve holdings of MBS are determined rather discretionary by the Fed and thus less endogenous than other variables used in counterfactual VAR analyses, for instance the term spread in Baumeister & Benati (2013) and Kapetanios et al. (2012).

5.4 Robustness analysis

Alternative model specifications with a different lag structure, more or less factors, and a different sample are now considered as part of the robustness analysis.¹⁵ Furthermore, alternative structural identification strategies are considered. Before I present the various robustness analyses, it is worth noting that the EM algorithm, as an alternative estimation method, can replicate the empirical findings of Bernanke et al. (2005) when a conventional monetary policy shock is under consideration, including the absence of the price puzzle that plagues low-dimensional VARs.¹⁶

With the ZLB, it makes little sense to consider shocks to the federal funds rate, and thus a reconciliation of the large body of empirical evidence of conventional monetary policy

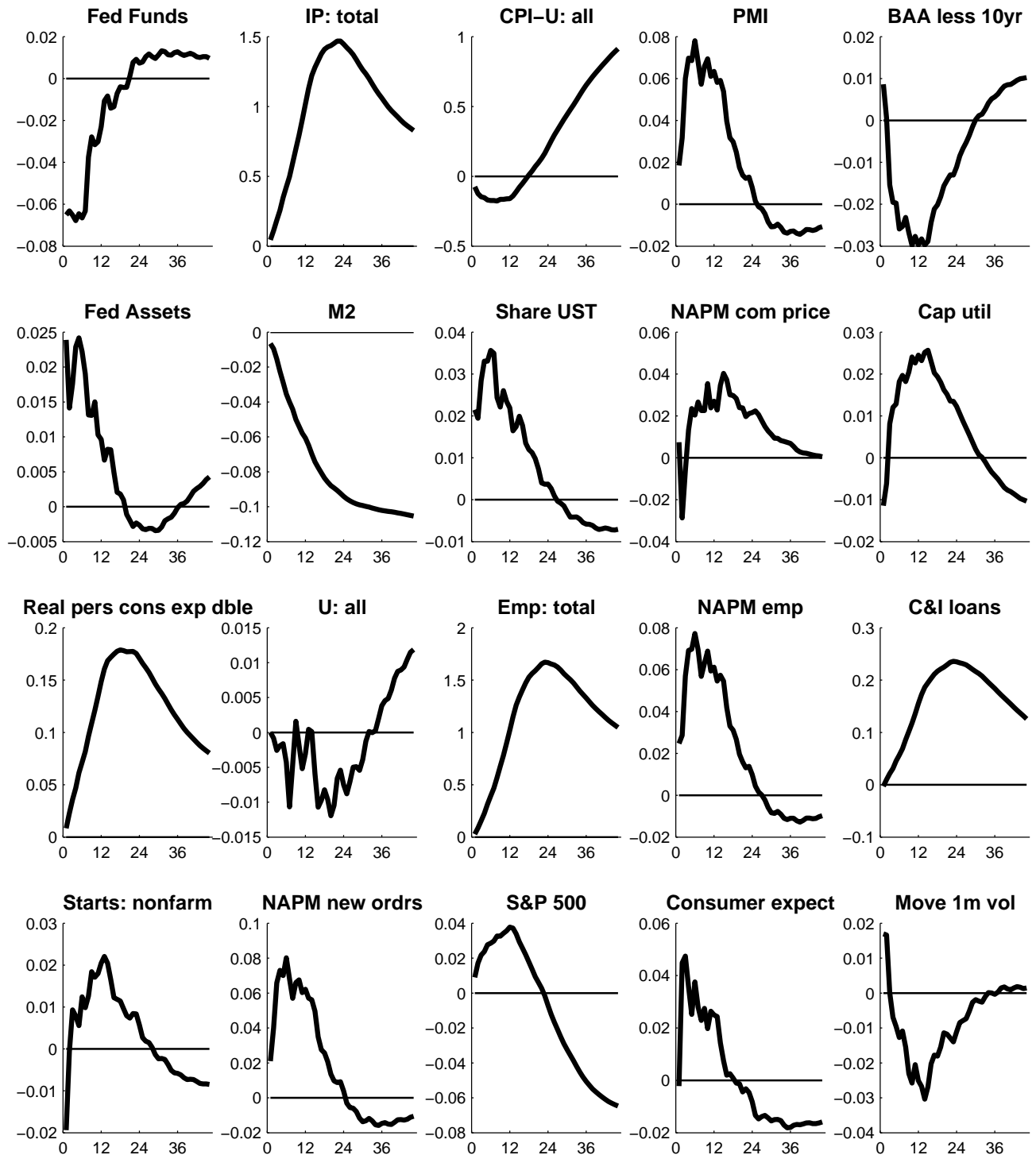
¹⁵The results of the robustness analysis are intended for an online appendix "B" and related figures have a prefix "B."

¹⁶They use a two-step principal component estimation procedure as well as Bayesian methods. To compare with their results, I estimate their preferred model specification with four factors and thirteen lags using exactly their dataset as well as an expanded sample up to the end of 2007 i.e. before the ZLB period. Results are very robust and available on request. See also Bork (2008) for a detailed discussion.

with the evidence of unconventional policy seems impossible. Researches have suggested replacing the federal funds rate by the so-called shadow rate which can be negative¹⁷ (see Bullard (2012) and Krippner (2013) to mention a only few). Although the use of the model-dependent shadow rate is debated among economists, it nevertheless allows yet another robustness check in which my unconventional policy results can be compared with those from structural shadow rate analyses. Consequently, I replace the federal funds rate in my panel with the shadow rate estimate in Wu & Xia (2014) and essentially update the findings of Bernanke et al. (2005). Figure 7 shows the responses to a expansionary shadow rate shock which are quite similar to the responses following an expansionary unconventional policy shock in Figure 5.¹⁸ In particular, the credit spread is reduced and the commercial and industrial loans (C&I loans) activity is increased, implying that the credit conditions seem to be improved. Moreover, industrial production, capacity utilization, inflation (CPI-U), and employment respond positively, while unemployment is reduced.

¹⁷The idea dates back to Black (1995), where he describes a method to calculate the value of the call option to hold cash at the zero lower bound. The call option value can be subtracted from the nominal rate (the federal funds rate) and may thus result in a negative shadow rate.

¹⁸To compare with the findings of Bernanke et al. (2005) and Wu & Xia (2014), the shadow rate shock is identified recursively.



The figure illustrates the impulse responses of key macroeconomic variables to an *expansionary* shock to the shadow federal funds rate identified by the **recursiveness** assumption. The shadow rate is from Wu & Xia (2014). The policy shock is here chosen to be expansionary to be comparable to the unconventional shocks in this paper. In order to compare with the empirical literature on conventional monetary policy, in particular Bernanke et al. (2005), one would have to multiply the impulse responses by minus one. Vertical axes are measured in standard deviations. Horizontal axes show time horizon.

Figure 7. Impulse responses to an expansionary shadow rate shock: Cholesky.

Lags. The impulse responses for the baseline model in Figure 5 are based on $p = 12$ lags because this specification implies absence of serial correlation of the VAR residuals. However, the results are robust when the number of lags is varied. For instance, the impulse responses generally remain the same if $p = 10$ or $p = 6$; see Figures C.1 and C.2 in Appendix C. However, residual autocorrelation becomes a problem when p is low. Specifically, the LM test of no residual autocorrelation is rejected for $p = 6$ but only marginally rejected for $p = 10$. Figure C.3 in Appendix C illustrates the residual autocorrelation for the baseline model and also the residual autocorrelation for the monetary policy factor for $p = 6$ and $p = 10$.

Sample. A shorter sample from 1959:01 to 2007:12 is considered as a robustness check of whether the structurally identifying assumptions in Table 1 are also able to identify conventional monetary policy shocks before the ZLB period. Conveniently, the weighted market share of UST and MBS would then only represent the market share of UST as the Federal Reserve did not hold MBS on its balance sheet. Accordingly, an increase in the UST share approximates an expansionary policy shock. The impulse responses in Figure C.4 in Appendix C are largely similar to the responses from an unconventional shock during the full sample, although the unrestricted response of the federal funds rate becomes positive after about a year. The responses should be interpreted with some caution, as a more precise measure of the market share should probably focus on the Federal Reserve's market share of T-bills to properly account for the subset of UST that the Federal Reserve uses in the conventional policy implementation.

Identifying assumptions. For the baseline model, one of the structurally identifying assumptions for an unconventional policy shock involved a positive response of the weighted market share of UST and MBS, cf. Table 1. Results are hardly distinguishable from the baseline model if this weighted market share is replaced with a non-weighted market share, as seen in Figure C.5 in Appendix C. I also replace the weighted market share with the simple UST market share. However, the simple measure is not a representative measure

of how the unconventional monetary policy was implemented during the recent crisis and the results from this particular identification strategy should probably be interpreted with caution. Figure C.6 in Appendix C shows that the responses are largely similar to the baseline responses but generally insignificantly different from zero.

6 Conclusion

This paper estimates the economy-wide responses of an unconventional monetary policy shock in terms of the Federal Reserve's large-scale asset purchases. Trillion dollar asset purchases of US treasuries and mortgage backed securities have been implemented since early 2009 with the purpose of improving long-term financing conditions. How effective are the LSAPs and what are the effects on the real economy? These are important research questions, and this paper contributes with positive empirical results, new data, and a new way of identifying unconventional policy shocks. Specifically, an unconventional monetary policy shock is identified as an increase in the Federal Reserve's holdings of US treasuries and mortgage backed securities that have a financial market impact, decreases the yield spread and credit spread, improves the financial market conditions, and increases inflation and measures of real activity. Note that the identification of an unconventional monetary policy shock is based on a relatively large number of identifying variables that achieves a more precise identification compared to the existing approaches in the literature, wherein a smaller set of identifying variables is typically used.

I find that an unconventional shock significantly drives down the long interest rate spread and the credit spread, and improves both the financial market conditions and the commercial and industrial loans activity. The impact on the real economy is significant: industrial production, capacity utilization, inflation, and employment have significantly positive responses, and unemployment is significantly reduced. The results are robust to alternative model specifications, and the results can be reconciled with the large body of

empirical evidence on conventional monetary policy if the federal funds rate is replaced with the shadow rate.

A counterfactual analysis based on the absence of the almost two trillion MBS purchases by the Federal Reserve Bank shows that a severe downturn was avoided. An almost 4% percentage point higher unemployment rate peaking at 12% seems to be avoided. Moreover, estimates shows that deflation, higher credit spreads, depressed lending, and heightened market volatility were also avoided.

In conclusion, this paper provides evidence that unconventional US monetary policy, as implemented via large-scale asset purchases, has significantly positive effects on credit conditions and leads to an economy-wide improvement of the real economy.

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A Data description

The sample is generally 1959:01 to 2014:09, except the last series starting with "CBOE SP 100 VOLATILITY INDEX". "Tcode" denotes transformation code: 1 means the level of x_t , 2 means Δx_t , 4 means $\ln x_t$, and 5 means $\ln x_t - \ln x_{t-1}$.

Table 2. List of US macroeconomic and financial time series.

	Variables			Tcode
1	INDL PROD - FINAL	Vol SA	2007=100	5
2	INDL PROD - TOTAL	Vol SA	2007=100	5
3	INDL PROD - CONSUMER GOODS	Vol SA	2007=100	5
4	INDL PROD - DURABLE CONSUMER GOODS	Vol SA	2007=100	5
5	INDL PROD - NONDURABLE CONSUMER GOODS	Vol SA	2007=100	5
6	INDL PROD - BUSINESS EQUIPMENT	Vol SA	2007=100	5
7	INDL PROD - MATERIALS, TOTAL	Vol SA	2007=100	5
8	INDL PROD - NONENERGY DURABLE GOODS MATL.	Vol SA	2007=100	5
9	INDL PROD - NONDURB GOODS MATL.	Vol SA	2007=100	5
10	INDL PROD - DURB MFG (IPD)	Vol SA	spliced,1992=100	5
11	INDL PROD - NONDURB MFG (IPD)	Vol SA	spliced,1992=100	5
12	INDL PROD - MINING (IPMIN)	Vol SA	spliced,1992=100	5
13	INDL PROD - UTILITIES (IPUT)	Vol SA	spliced,1992=100	5
14	INDL PROD - MANUFACTURING (SIC)	Vol SA	2007=100	5
15	INDL PROD - TOTAL INDEX	Vol SA	2007=100	5
16	US INDL UTILIZATION - MANUFACTURING (SIC)	SA		1
17	US ISM PURCHASING MANAGERS INDEX	SA		1
18	US ISM MANUFACTURERS SURVEY: PRODUCTION	SA		1
19	US PERS INCOME, REAL (AR) (BCI 52)	Cnst price SA	2009 PRICE	5
20	US PERS INCOME LESS TRANSFER PAYMENTS	Cnst price SA	2009 PRICE	5
21	US TOTAL CIVILIAN EMPLOYMENT	Vol SA		5
22	US EMPLOYED, NONFARM - (16 YRS+)	Vol SA		5
23	US UNEMPLOYMENT RATE	SA		1
24	US AVERAGE DURATION OF UNEMPL. (WEEKS)	Vol SA		1
25	US UNEMPLOYED FOR LESS THAN 5 WEEKS	Vol SA		1
26	US UNEMPLOYED FOR 5 TO 14 WEEKS	Vol SA		1
27	US UNEMPLOYED FOR 15 WEEKS OR MORE	Vol SA		1
28	US UNEMPLOYED FOR 15 TO 26 WEEKS	Vol SA		1
29	US EMPLOYED - NONFARM INDUSTRIES TOTAL	Vol SA		5
30	US EMPLOYED - TOTAL PRIVATE	Vol SA		5
31	US EMPLOYED - GOODS-PRODUCING	Vol SA		5
32	US EMPLOYED - NAT RESOURCES AND MINING	Vol SA		5
33	US EMPLOYED - CONSTRUCTION	Vol SA		5
34	US EMPLOYED - MANUFACTURING	Vol SA		5
35	US EMPLOYED - DURABLE GOODS	Vol SA		5
36	US EMPLOYED - NONDURABLE GOODS	Vol SA		5
37	US EMPLOYED - SERVICE-PROVIDING	Vol SA		5
38	US EMPLOYED - TRADE, TRANSP., AND UTILITIES	Vol SA		5
39	US EMPLOYED - WHOLESALE TRADE	Vol SA		5
40	US EMPLOYED - FINANCIAL ACTIVITIES	Vol SA		5
41	US EMPLOYED - PRIVATE SERVICE-PROVIDING	Vol SA		5
42	US EMPLOYED - GOVERNMENT	Vol SA		5
43	US AVG WKLY HOURS - MANUFACTURING	Vol SA		1
44	US AVG OVERTIME HOURS - MANUFACTURING	Vol SA		1
45	US ISM MANUFACTURERS SURVEY: EMPLOYMENT	SA		1
46	US PERS CONSUMPTION EXPEND (AR)	Curr price SA		5
47	US PERS CONSUMPTION EXPEND - DURB (AR)	Curr price SA		5
48	US PERS CONSUMPTION EXPEND - NONDURB (AR)	Curr price SA		5
49	US PERS CONSUMPTION EXPEND - SERVICES (AR)	Curr price SA		5
50	US PCE: DURB - NEW AUTOS	Curr price SA		5
51	US NEW PRIV HOUSING STARTED	Vol SA		4
52	US HOUSING STARTED - NORTHEAST (AR)	Vol SA		4
53	US HOUSING STARTED - MIDWEST (AR)	Vol SA		4
54	US HOUSING STARTED - SOUTH (AR)	Vol SA		4

Continued on next page

	Variables			Tcode
55	US HOUSING STARTED - WEST (AR)	Vol SA		4
56	US BUILD PERMITS TO NEW PRIV HOUSING	Vol SA		4
57	US ISM MANUFACTURERS SURVEY: INVENT	Not SA		1
58	US ISM MANUFACTURERS SURVEY: NEW ORDERS	SA		1
59	US ISM MANUFACTURERS SURVEY: DELIVERIES	SA		1
60	US NEW ORDERS - CONSUMER GOODS (BCI 8)	Cnst price SA	1982 PRICES	5
61	US NEW ORDERS - NONDEFENSE CAP (BCI 27)	Cnst price SA	1982 PRICES	5
62	NYSE STOCK PRICE INDEX		spliced,1965=100	5
63	US SP COMPOSITE INDEX (EP)		(1941-43=100)	5
64	SP 500 STOCK PRICE INDEX: INDUSTRIALS		spliced	5
65	US SP 500 COMPOSITE - DIVIDEND YLD			1
66	US SP 500 COMPOSITE - REAL P/E RATIO			1
67	SW SWISS FRANCS TO USD			5
68	JP JAPANESE YEN TO US USD	Interest Rates		5
69	UK US USD TO 1	Interest Rates		5
70	CN EXCHANGE RATE: CURRENCY PER USD	Not SA	USD/CAD	5
71	US FEDERAL FUNDS RATE (AVG.)			1
72	US T-BILL 3 MONTH	Interest Rates		1
73	US T-BILL 6 MONTH	Interest Rates		1
74	US TREASURY YIELD CONST. MAT. - 1 YEAR	Interest Rates		1
75	US TREASURY YIELD CONST. MAT. - 5 YEAR	Interest Rates		1
76	US TREASURY YIELD CONST. MAT. - 10 YEAR	Interest Rates		1
77	US CORPORATE BOND YIELD - MOODYS AAA	Interest Rates	AVRGE	1
78	US CORPORATE BOND YIELD - MOODYS BAA	Interest Rates	AVRGE	1
79	US T-BILL 3 MONTH	Interest Rates		1
80	US T-BILL 6 MONTH	Interest Rates		1
81	US TREASURY YIELD CONST. MAT. - 1 YEAR	Interest Rates		1
82	US TREASURY YIELD CONST. MAT. - 5 YEAR	Interest Rates		1
83	US TREASURY YIELD CONST. MAT. - 10 YEAR	Interest Rates		1
84	US CORPORATE BOND YIELD - MOODYS AAA	Interest Rates	AVRGE	1
85	US CORPORATE BOND YIELD - MOODYS BAA	Interest Rates	AVRGE	1
86	CHANGE 10YR LESS CHANGE IN FED FUNDS	Interest Rates		1
87	CHANGE 10YR GOV. BOND	Interest Rates		1
88	BAA - 10YR SPREAD	Interest Rates		1
89	US MONEY SUPPLY M1	Curr price SA		5
90	US MONEY SUPPLY M2	Curr price SA		5
91	US MONEY SUPPLY M2 (BCI 106)	Cnst price SA	2009 PRICE	5
92	US MONETARY BASE CURN	Curr price NSA		5
93	US COML AND INDL. LOANS OUTSTAND.	Cnst price SA	2009 PRICE	5
94	US COML BANK ASSETS-COML. AND INDL LOANS	SA		1
95	US NONREVLV CONS CREDIT OUTSTAND	Curr price SA		5
96	OUTSTANDING MORTGAGE DEBT	yearly growth		1
97	FRB ASSETS: UST, MORTG., FDRL. AGENCY	yearly growth		1
98	WGT FED SHARE UST AND MBS	yearly change		1
99	FED SHARE UST	yearly change		1
100	FED SHARE UST AND MBS	yearly change		1
101	US ISM MANUFACTURERS SURVEY: PRICES PAID	SA		1
102	US PPI - FINISHED GOODS	Price index SA	1982=100	5
103	US PPI - FINISHED CONSUMER GOODS	Price index SA	1982=100	5
104	US PPI - INTERMED. MATL, SUPPL AND COMP	Price index SA	1982=100	5
105	US PPI - CRUDE MATERIALS	Price index SA	1982=100	5
106	US CPI - ALL URBAN: ALL ITEMS	Price index SA	1982-1984=100	5
107	US CPI - APPAREL	Price index SA	1982-1984=100	5
108	US CPI - TRANSPORTATION	Price index SA	1982-1984=100	5
109	US CPI - MEDICAL CARE	Price index SA	1982-1984=100	5
110	US CPI - COMMODITIES	Price index SA	1982-1984=100	5
111	US CPI - DURB	Price index SA	1982-1984=100	5
112	US CPI - SERVICES	Price index SA	1982-1984=100	5
113	US CPI - ALL ITEMS LESS FOOD	Price index SA	1982-1984=100	5
114	US CPI - ALL ITEMS LESS SHELTER	Price index SA	1982-1984=100	5
115	US CPI - ALL ITEMS LESS MEDICAL CARE	Price index SA	1982-1984=100	5
116	US AVG HRLY EARN - CONSTRUCTION	Curr price SA	USD/Hour	5
117	US AVG HRLY EARN - MANUFACTURING	Curr price SA	USD/Hour	5
118	US CONSUMER CONFIDENCE - EXPECTATIONS	Price index SA	1966M1=100	1
119	MODEL AVG VOL EST		Garch(1,1)	1

Continued on next page

	Variables		Tcode
120	CBOE SP 100 VOLATILITY INDEX	1986:06 -	1
121	FRB CHICAGO NAT FIN CONDITION INDEX	1973:01-	1
122	KANSAS CITY FINANCIAL STRESS INDEX	1990:02-	1
123	TED SPREAD	1986:01-	1
124	US AVG CONS EXPECT FOR BUS COND. (INV)	1978:02-	1
125	ML MOVE BOND VOLATILITY INDEX	1988:06-	1
126	ADS BUS CONDITION INDEX (INV)	1960:03-	1

Notes: Proprietary data sources: Datastream. Public sources: Federal Reserve Economic Data (FRED). Federal Housing Finance Agency.

B Zero and sign restrictions in a dynamic factor model

The algorithm of Arias et al. (2014) for imposing zero and sign restrictions on the impulse responses from VARs are now slightly modified to allow for similar restrictions on DFMs. Arias et al. (2014) extend the efficient sign restriction algorithm of Rubio-Ramírez et al. (2010) to allow for zero restrictions, and ARW forcefully argue that whenever sign restrictions are combined with zero restrictions, it becomes crucial to condition the draws on the zero restrictions. Otherwise, additional sign restrictions are implicitly imposed.

The notation is first presented and then the algorithm. The number of variables in the VAR is r and p denotes the lags. The structural VAR parameters generally carry the letter A , with A_0 denoting the impact matrix and A_1, \dots, A_p denoting the autoregressive parameters, while the structural shocks are mean zero and the variance is $E[\varepsilon_t \varepsilon_t^\top] = I_r$. The reduced form parameters are then $\Phi_1 = A_0^{-1}A_1, \dots, \Phi_p = A_0^{-1}A_p$ as seen in (1) with the reduced form residuals u_t characterized by $E[u_t u_t^\top] = A_0^{-1} (A_0^{-1})^\top = \Sigma_u$. The reduced form companion matrix without any subscript, Φ , is of dimension $rp \times rp$.

The impulse response of the i th variable (factor) in the VAR to the j th structural shock at horizon h is seen in row i , column j in Θ_h from the structural VMA representation¹⁹:

$$F_t = \sum_{h=1}^{\infty} \Theta_h \varepsilon_{t-h} = J \Phi^h J^\top \mathcal{A}_0^{-1} \quad (11)$$

where J is an $rp \times r$ selection matrix with zeros except for the top $r \times r$ matrix, which is I_r , and where Φ^h is the companion matrix to the power of h . The response of the n th observed variable to the j th structural shock at horizon h is seen in row n , column j in Ψ_h :

$$X_t = \sum_{h=1}^{\infty} \Psi_h \varepsilon_{t-h} = \Lambda J^\top \Phi^h J \mathcal{A}_0^{-1} \quad (12)$$

Consider restricting the VAR impulse responses in (11) by zero and sign restrictions

¹⁹ Θ_i obeys a recursion; see Lütkepohl (2005) chapter 2 for the details.

for a maximum of h periods and stack the responses in the $rh \times r$ dimensional matrix $F(\mathcal{A})$, where A emphasizes the structural parameters. Alternatively, a subset $r \leq \rho \leq N$ of the panel impulse responses in (12) are stacked in the $\rho h \times r$ matrix $X(\mathcal{A})$. The z_j zero restrictions imposed on the j th structural shock are represented by matrices Z_j , whereas the s_j sign restrictions are represented by the matrix S_j . Thus, the zero restrictions are satisfied if $Z_j F(\mathcal{A}) e_j = 0$, where e_j is the j th column of I_r ; alternatively for an appropriate Z_j , $Z_j X(\mathcal{A}) e_j = 0$. Similarly, for the sign restrictions, $S_j F(\mathcal{A}) e_j > 0$. Note that the impulse responses are only set identified, as any orthogonal $r \times r$ matrix Q , $S_j F(\mathcal{A}Q) e_j > 0$ will also satisfy the restrictions.

The idea is to generate a large number of orthogonal matrices Q , but where the generation of these matrices takes into account the zero restrictions. Any Q that do not satisfy the zero and sign restrictions will be discarded, while the Q that satisfy the restrictions will be kept:

$$\begin{aligned} Z_j \mathcal{F}(\mathcal{A}Q) e_j &= Z_j \mathcal{F}(\mathcal{A}) Q e_j = Z_j \mathcal{F}(\mathcal{A}) q_j = 0 \\ S_j \mathcal{F}(\mathcal{A}Q) e_j &= S_j \mathcal{F}(\mathcal{A}) q_j > 0 \end{aligned} \tag{13}$$

or for appropriate Z_j and S_j

$$\begin{aligned} Z_j \mathcal{X}(\mathcal{A}Q) e_j &= Z_j \mathcal{X}(\mathcal{A}) q_j = 0 \\ S_j \mathcal{X}(\mathcal{A}Q) e_j &= S_j \mathcal{X}(\mathcal{A}) q_j > 0 \end{aligned} \tag{14}$$

The following algorithm from ARW shows how to draw the structural parameters A conditional on zero and sign restrictions. Without loss of generality, the zero restrictions are imposed on the first k variables, where $1 \leq k \leq r$ and z_j need to satisfy $z_j \leq n - k$. The first step in the algorithm is then to obtain the $r \times k$ matrix Q_k corresponding to the zero restrictions and then conditional on Q_k to draw the remaining Q_{n-k} matrix, such that for each draw $Q = \begin{bmatrix} Q_k & Q_{n-k} \end{bmatrix}$.

Algorithm 1 *Given the reduced form VAR parameters (Φ, Σ_u) and a mapping $\hat{h}(\Phi, \Sigma_u, I_r)$*

to the structural parameters A , where \hat{h} could involve computing the Cholesky decomposition or a symmetric and positive definite square root matrix, the Q_k part of Q is first drawn and secondly the remaining Q_{n-k} .

1. Apply a Gibbs sampler to draw from the uniform distribution of Q_k conditional on $Z_j F(\mathcal{A}) q_j = 0$ for $1 \leq j \leq k$. Given the structural parameters in A and $q_1^{(i+1)}, \dots, q_{j-1}^{(i+1)}, q_{j+1}^{(i)}, \dots, q_k^{(i)}$:

(a) Let $\hat{N}_j^{(i+1)}$ denote the orthonormal basis for the null space of

$$\begin{bmatrix} q_1^{(i+1)} & \cdots & q_{j-1}^{(i+1)} & [Z_j \mathcal{F}(\mathcal{A})]^\top & q_{j+1}^{(i)} & \cdots & q_k^{(i)} \end{bmatrix}^\top$$

(b) Upon drawing $\tilde{x}_j^{(i+1)}$ from a standard normal, compute $q_j^{(i+1)}$ from

$$q_j^{(i+1)} = \frac{\begin{bmatrix} \hat{N}_j^{(i+1)} \mathbf{0}_{r, r - \hat{r}_j^{(i+1)}} \end{bmatrix} \tilde{x}_j^{(i+1)}}{\left\| \begin{bmatrix} \hat{N}_j^{(i+1)} \mathbf{0}_{r, r - \hat{r}_j^{(i+1)}} \end{bmatrix} \tilde{x}_j^{(i+1)} \right\|}$$

where $\hat{r}_j^{(i+1)}$ is the rank of $\hat{N}_j^{(i+1)}$.

2. Step 1a to 1b are repeated L times to ensure that $q_1^{(L)}, \dots, q_k^{(L)}$ converges to the uniform distribution of Q_k conditional on $Z_j F(\mathcal{A}) q_j = 0$. The convergence is faster if the Gibbs sampler is filled in with the following starting values. Specifically, $\tilde{N}_j^{(0)}$ is the orthonormal basis of the null space of

$$\begin{bmatrix} q_1^{(0)} & \cdots & q_{j-1}^{(0)} & [Z_j \mathcal{F}(\mathcal{A})]^\top \end{bmatrix}^\top$$

and upon drawing $\tilde{x}_j^{(0)}$, starting values for the q 's can be calculated from:

$$q_j^{(0)} = \frac{\begin{bmatrix} \tilde{N}_j^{(0)} \mathbf{0}_{r, r - \tilde{r}_j^{(0)}} \end{bmatrix} \tilde{x}_j^{(0)}}{\left\| \begin{bmatrix} \tilde{N}_j^{(0)} \mathbf{0}_{r, r - \tilde{r}_j^{(0)}} \end{bmatrix} \tilde{x}_j^{(0)} \right\|}$$

Remark 2 Alternatively, $[Z_j \mathcal{F}(\mathcal{A})]^\top$ corresponding to (13) is replaced by $[\Lambda Z_j \mathcal{F}(\mathcal{A})]^\top$ corresponding to (14).

3. Draw the remaining Q_{n-k} conditional on Q_k constructed from the converged $q_1^{(L)}, \dots, q_k^{(L)}$. Let \tilde{X}_{r-k} be a $(r-k) \times (r-k)$ random matrix where each element is iid $\sim N(0, 1)$. Let N_k denote the orthonormal basis for the null space of Q_k and compute the QR decomposition of $N_k \tilde{X}_{r-k}$, i.e.

$$\begin{aligned} QR &= N_k \tilde{X}_{r-k} \\ Q_{n-k} &= Q \end{aligned}$$

from which the complete orthogonal matrix Q can be assembled as $Q = \begin{bmatrix} Q_k & Q_{n-k} \end{bmatrix}$.

4. Keep the draw if $S_j F(\mathcal{A}) q_j > 0$ for all structural shocks identified with sign restrictions. Note, the zero restrictions are already satisfied.

Remark 3 Again, $S_j F(\mathcal{A}) q_j > 0$ may be replaced by $S_j X(\mathcal{A}) q_j > 0$.

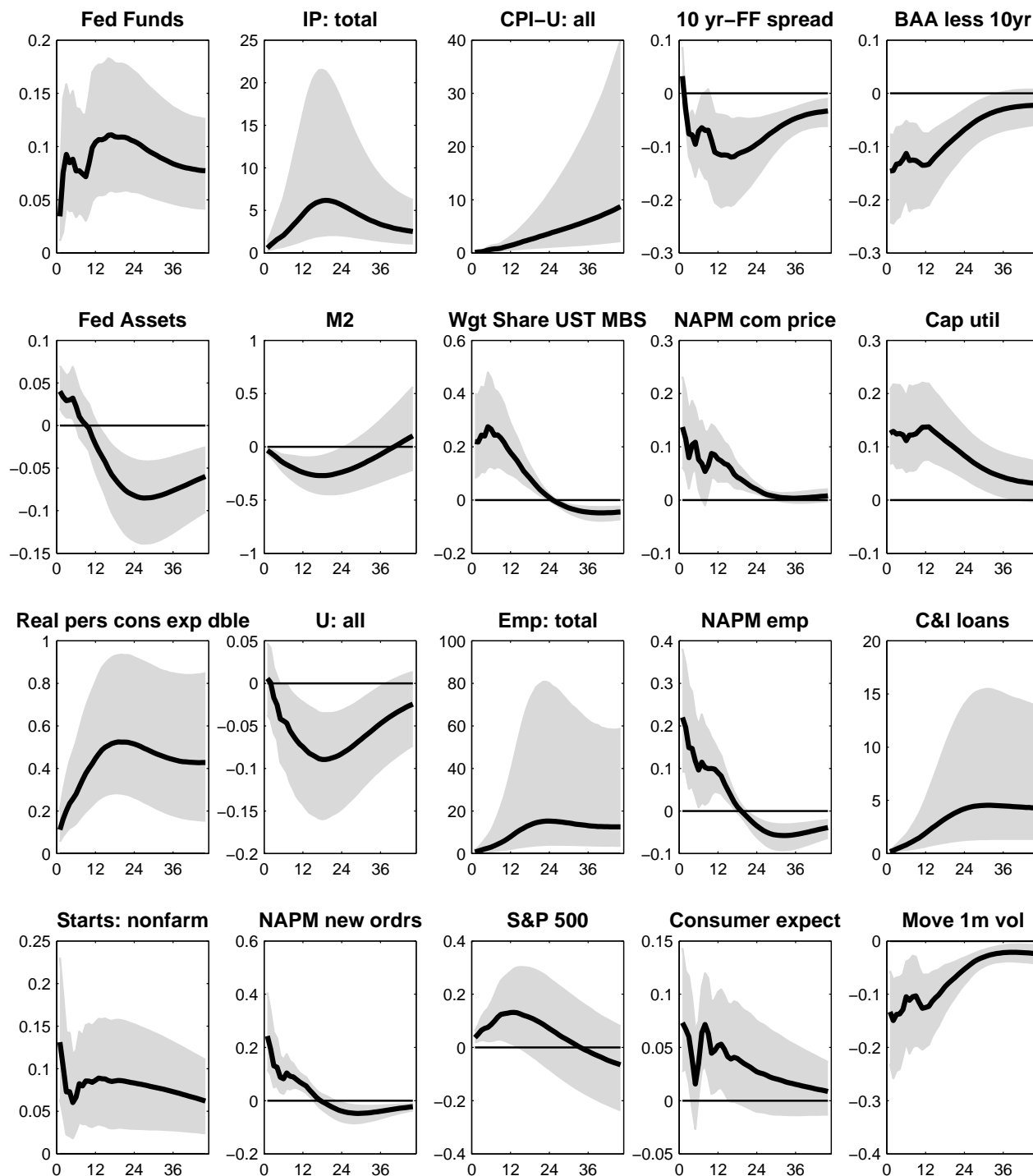
5. Return to step 1 and repeat until the desired number of draws has been obtained.

C Supplementary material

Table C.1. Summary of important QE announcements by the Federal Reserve

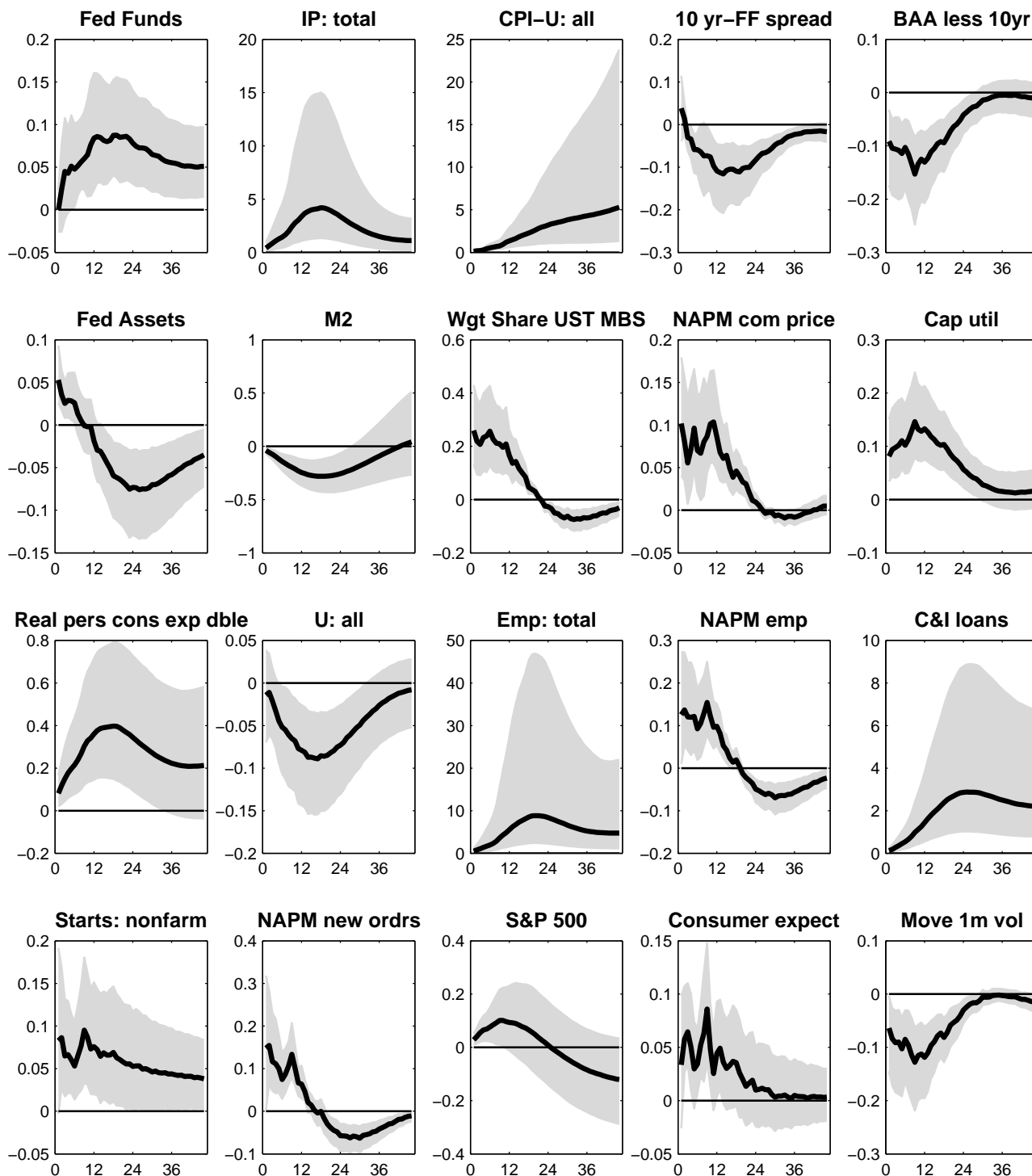
Date	QE	Description of event
November 25, 2008	QE1	LSAP announcement. Fed will purchase up to 100 bn. GSE and 500 bn. MBS
December 1, 2008	QE1	Bernanke: Could purchase UST og MBS in substantial quantities.
December 16, 2008	QE1	Federal funds rate cut. Fed suggests to extend QE to USTs
January 28, 2009	QE1	Fed stands ready to expand QE by buying USTs
March 18, 2009	QE1	The Fed will purchase 300 bn. in USTs and additional 750 bn. MBS and 100 bn. GSE
September 23, 2009	QE1	Agency debt and MBS will end 2010:Q1
November 4, 2009	QE1	Agency debt purchases downsized to 175 bn.
August 10, 2010	QE2	The Fed will reinvest principal payments form LSAPs in USTs
August 27, 2010	QE2	Bernanke: "role for additional QE should further action prove necessary".
September 21, 2010	QE2	FOMC statement: Additional accommodation if needed
November 3, 2010	QE2	Fed will buy 600 bn. of UST with 75 bn./month
September 21, 2011	Twist	Fed will purchase 400 bn. of long UST and sell short UST
June 20, 2012	Twist	Fed continues with operation Twist (MEP) in 2012 with 45 bn. UST/month
September 13, 2012	QE3	Fed will purchase 85 bn. of UST in addition to 40 bn. MBS per month
December 12, 2012	QE3	Fed will purchase 45 bn. of UST in addition to 40 bn. MBS per month
December 18, 2013	QE3	Fed cuts purchases of MBS by 10 bn. to 75 bn.

Figure C.1. Robustness analysis: Impulse responses to an unconventional monetary policy shock with 6 lags.



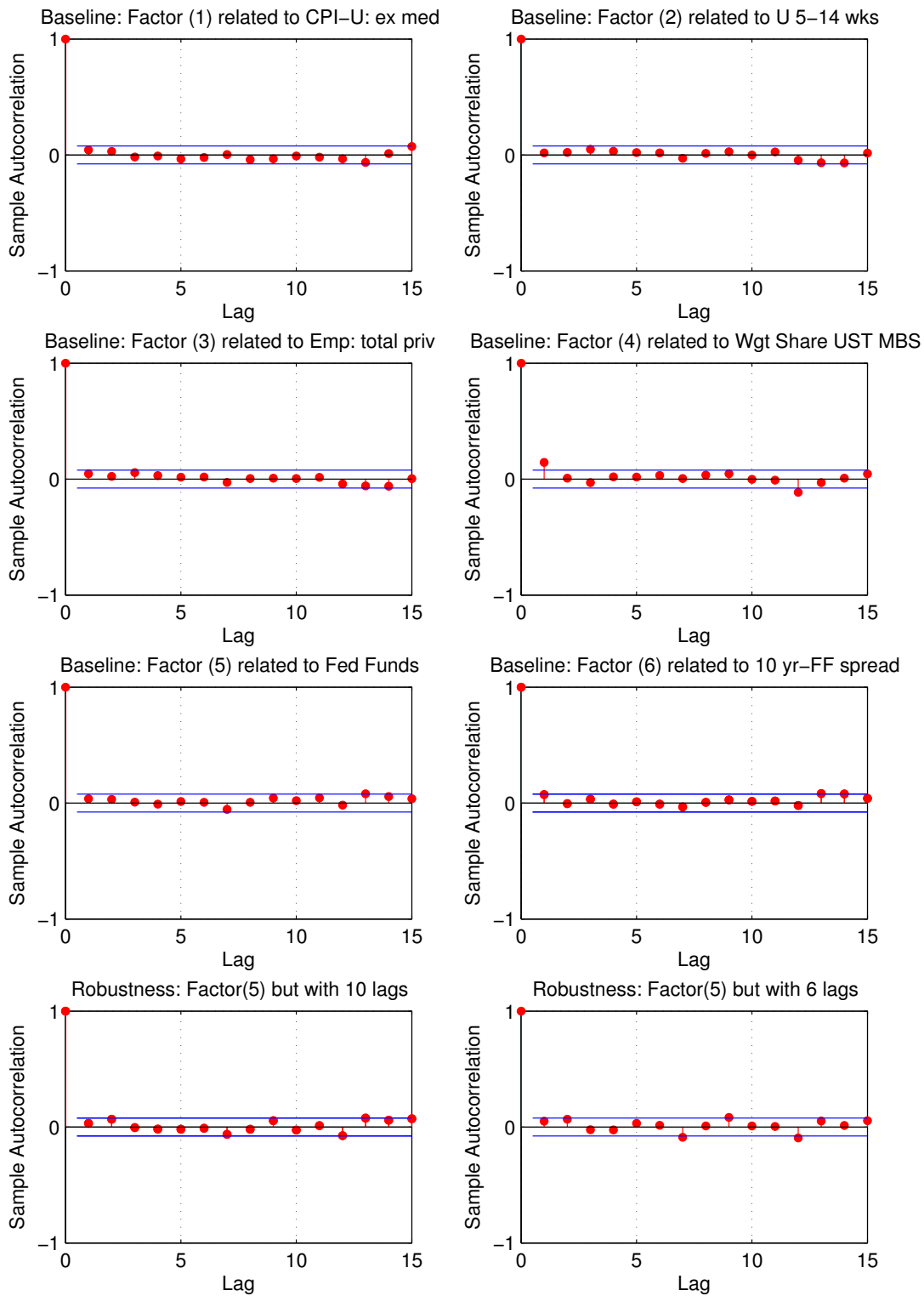
The figure illustrates the impulse responses of key macroeconomic variables to a positive shock to unconventional monetary policy identified by **zero and sign restrictions**. All responses are normalized by considering a one percent change in the innovations to the market share (Wgt Share). Vertical axes are measured in standard deviations. Horizontal axes show time horizon. The 68% confidence intervals are shaded and based on the first 10^4 satisfied draws out of a total of 10^6 generated draws.

Figure C.2. Robustness analysis: Impulse responses to an unconventional monetary policy shock with 10 lags.



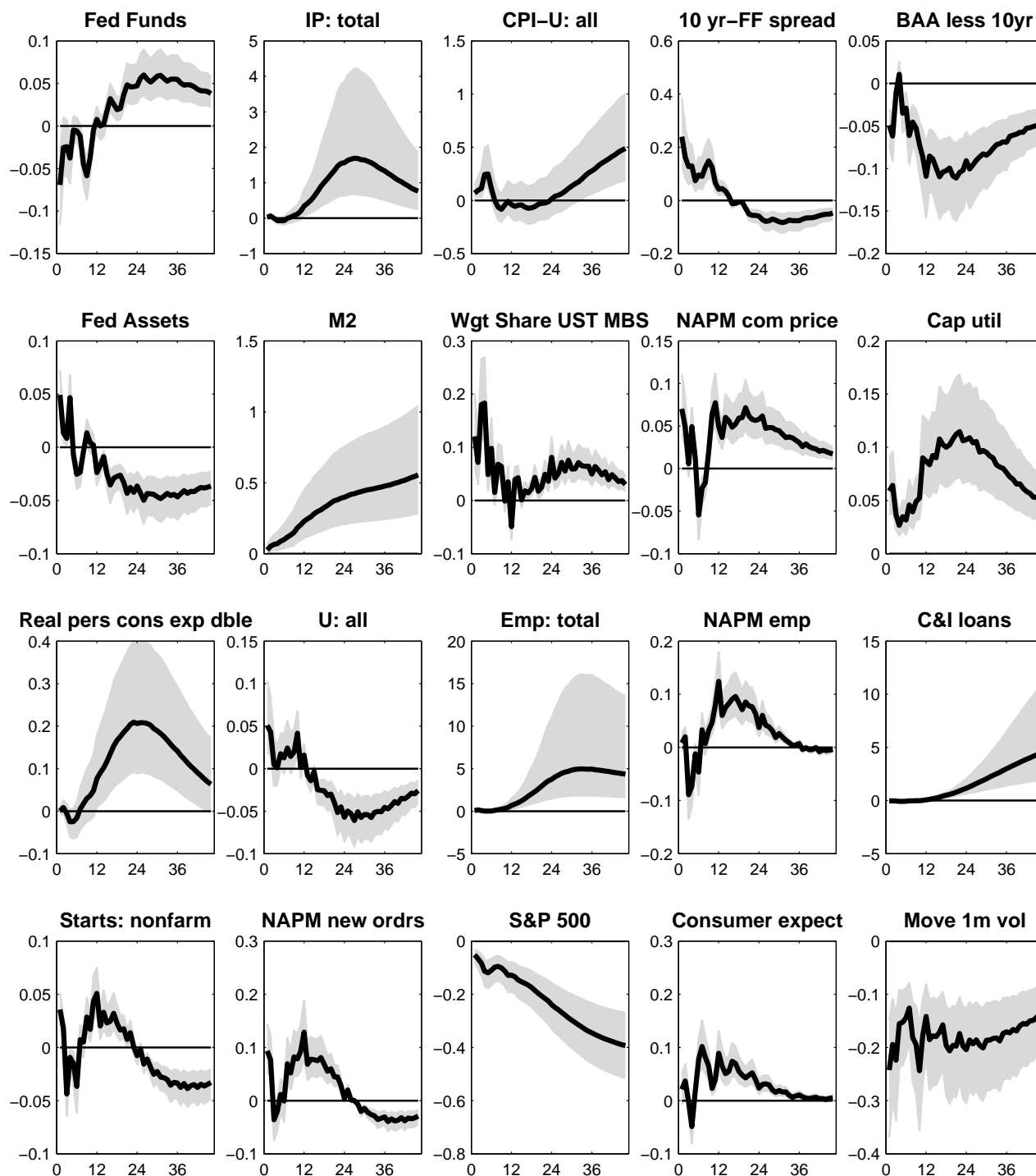
The figure illustrates the impulse responses of key macroeconomic variables to a positive shock to unconventional monetary policy identified by **zero and sign restrictions**. All responses are normalized by considering a one percent change in the innovations to the market share (Wgt Share). Vertical axes are measured in standard deviations. Horizontal axes show time horizon. The 68% confidence intervals are shaded and based on the first 10^4 satisfied draws out of a total of 10^6 generated draws.

Figure C.3. Robustness analysis: Residual autocorrelation.



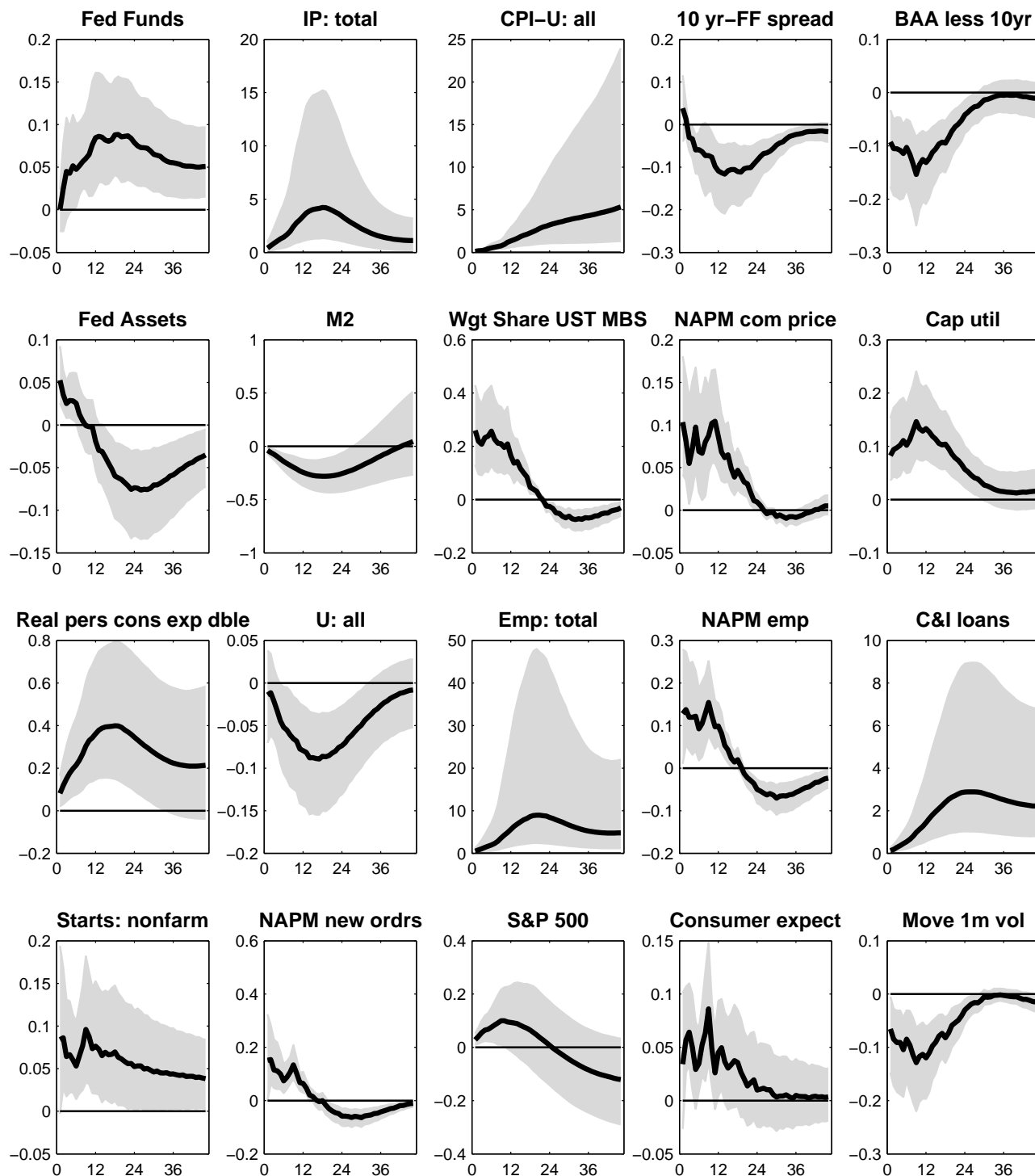
The figure plots the autocorrelation of the VAR residuals of the dynamic factor model. The upper six plots represent the baseline model for each of the six factors. As a robustness analysis, the lower two plots show the residual autocorrelation for the monetary policy factor in an alternative model specification with $p = 6$ and $p = 10$ lags, respectively.

Figure C.4. Robustness analysis: Impulse responses to an unconventional monetary policy shock during 1959-2007.



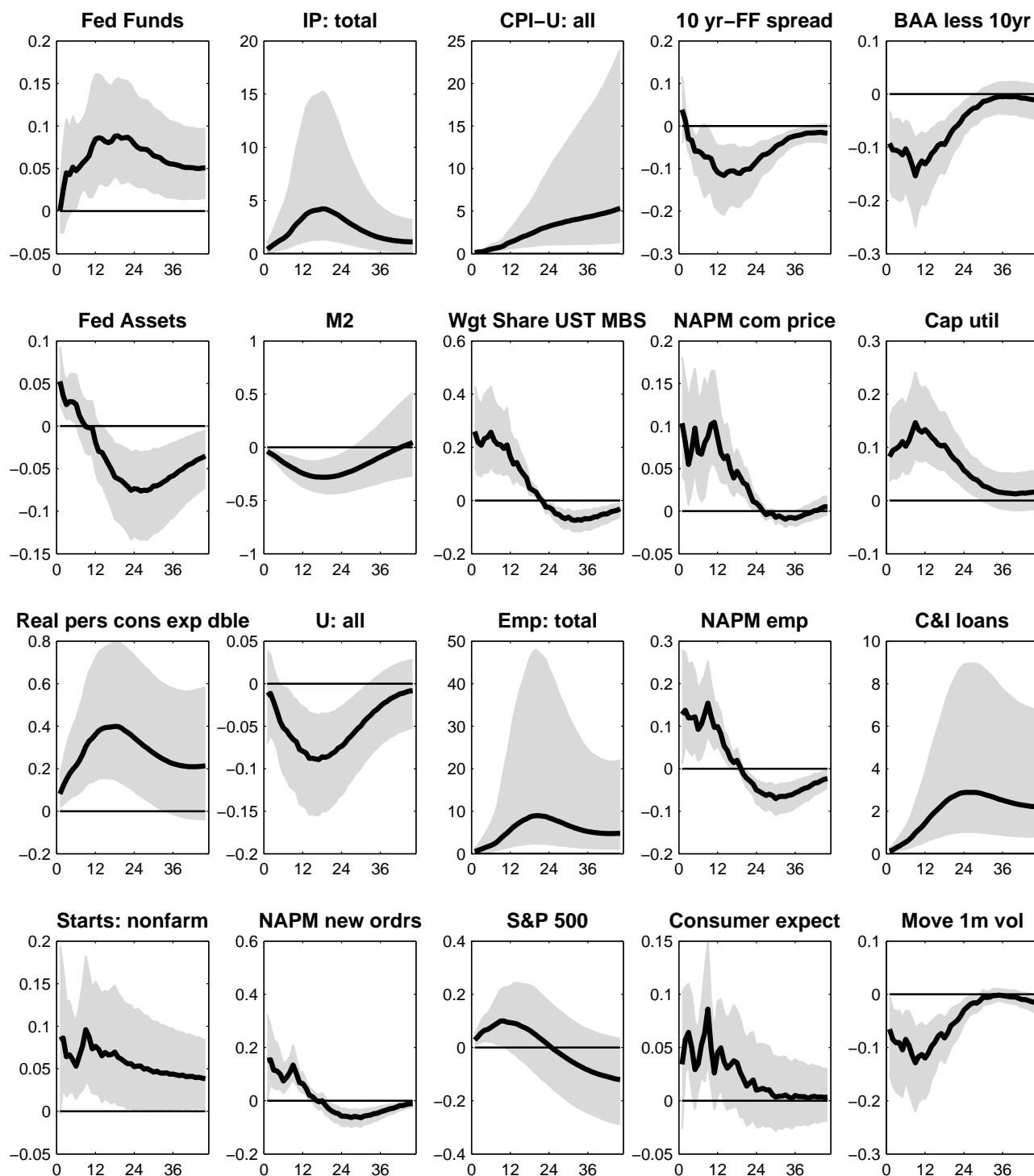
The figure illustrates the impulse responses of key macroeconomic variables to a positive shock to unconventional monetary policy identified by **zero and sign restrictions**. The sample is reduced to 1959-2007. All responses are normalized by considering a one percent change in the innovations to the market share (Wgt Share). Vertical axes are measured in standard deviations. Horizontal axes show time horizon. The 68% confidence intervals are shaded and based on the first 10^4 satisfied draws out of a total of 10^6 generated draws.

Figure C.5. Robustness analysis: Impulse responses to an unconventional monetary policy shock using the non-weighted market share.



The figure illustrates the impulse responses of key macroeconomic variables to a positive shock to unconventional monetary policy identified by **zero and sign restrictions**. In this figure, the weighted market share is replaced by the non-weighted market share. All responses are normalized by considering a one percent change in the innovations to the market share. Vertical axes are measured in standard deviations. Horizontal axes show time horizon. The 68% confidence intervals are shaded and based on the first 10^4 satisfied draws out of a total of 10^6 generated draws.

Figure C.6. Robustness analysis: Impulse responses to an unconventional monetary policy shock using the UST market share only.



The figure illustrates the impulse responses of key macroeconomic variables to a positive shock to unconventional monetary policy identified by **zero and sign restrictions**. In this figure, the weighted market share is replaced by the non-weighted market share. All responses are normalized by considering a one percent change in the innovations to the market share. Vertical axes are measured in standard deviations. Horizontal axes show time horizon. The 68% confidence intervals are shaded and based on the first 10^4 satisfied draws out of a total of 10^6 generated draws.